Strategic Real Options: Capacity Optimization and Demand Structures

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PROEFSCHRIFT

ter verkrijging van de graad van doctor aan Tilburg University op gezag van de rector magnificus, prof.dr. Ph. Eijlander, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op

vrijdag 5 december 2014 om 14.15 uur

door

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geboren op 12 augustus 1987 te Goes.

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Acknowledgements

This book could not have been written without the help and support from many others. Let me start by thanking the two most important contributors: Peter Kort and Verena Hagspiel. I count myself fortunate to have the two of you as my supervisors. You have always supported and motivated me in the previous years, and at the same time allowed me to make considered choices when needed. Peter, my weekly supervisor, you introduced me to the academical world by proposing to write my master theses together with you and Verena. After that, the door of your office has always been open for questions, and responds to emails came in general within minutes. Your readiness and active involvement is very much appreciated! Thank you for carrying on your enthusiasm for this area of research to me. Verena, while Peter was based in Tilburg, you lived and traveled all around Europe during the three years of my PhD. Nevertheless, every month you took the time and effort to come to the Netherlands. Of course, I'd like to believe that the main ground for this is my supervision, but Michiel could have been this little extra incentive. At the times that you were not physically in Tilburg, you were always available via skype, of which I thankfully made use. I am indebted to you for your very thorough reviews of my papers, which really helped me to grow in academic writing. I would also like to thank you for hosting me in Trondheim, I have had a great time!

In addition, I am honored to have Stein-Erik Fleten, Kuno Huisman, Cláudia Nunes and Dolf Talman in my committee. Thank you for taking the time to carefully read my thesis, giving some valuable feedback, and coming to Tilburg (twice!) for this purpose. I would also like to take this opportunity to thank Afzal

Siddiqui, who hosted me for three months at the University College London. You, together with the post-docs and PhD students at the statistical department, really made me feel welcome. I have learned a lot from you, for which I am truly grateful. Some relatively new members of the Real Options Research Group (RORG), and my fellow PhD students, are Nick Huberts and Maria Lavrutich. Thank you for the fun and helpful RORG meetings on Tuesday mornings. To you goes the (questionable) honour that by the end of my PhD, I am finally able to correctly pronounce 'oligopoly'.

Besides the Real Options group, there were some other people that made these three years of research especially pleasant for me. Let me start by mentioning my lovely office mates. Noelia, I was a lucky person that I could share my office with you for the first two years. Obviously, Tilburg was not exciting enough for you, so moving to Utrecht was the most feasible option such that I could still regularly enjoy your presence in Tilburg. Thank you for introducing me to many of your friends at the great parties you threw. Again I was lucky when Aida, also known as my Spanish professor or otherwise as my Dutch student, became my new office mate. I enjoyed our weekly language-exchange diners, and the Spanish crash course weekends I spent with you welcoming family in Llavaneres. Mitzi, our un-official office mate, you delighted our office with frequent visits and just like Noelia and Aida you have become a good friend of mine. In addition, I would like to thank the friends that I made during lunch. The interesting and often funny lunch conversations really helped to make me feel at home at the university and relax in between the working hours.

Also outside the university, friends and family have been really supportive by simply being a friend. Marlou, Nadine and Isabel, you belong to both categories. Whether it is painting houses, fumbling around flee markets or lazy sleep-overs, I enjoy doing all of this with you! I would also like to mention my high school friends from Zeeland, thank you for investing in our friendship for almost 10 years already. I always look forward to our dinners, home-made high teas, crafty gatherings and the game nights in Breda. Thinking about Tilburg, I think about the 'Toetjesclub', the foundation place of my dear group of friends: Marieke, Marjolein, Marloes, Myrthe and Yvette. Good excuses to meet (and eat) were created over the years and became traditions: the yearly Tilburg-zingt barbecue, toetjesclub-weekend, or new-years-eve dinner. I hope that we can continue these traditions for the many years to come. Myrthe, you know you impress me with your persistence in either whitening my colorful house or joining me on an amazing biking trip to Paris. Marieke, I also want to thank you for the unforgettable time that we spend together during the holiday at the start of our professional life. I am lucky to have you as my friends, among with other friends that I did not mention.

Last, but definitely not least, I could not have finished this dissertation without the support from my parents, brothers and sister. For expressing my gratitude towards them I will continue in Dutch. Lieve papa en mama, hier is dan de mogelijkheid om jullie zwart op wit te bedanken voor de onvoorwaardelijke hulp en steun afgelopen jaren, zowel gedurende mijn PhD als tijdens mijn studie. Jullie hebben altijd met mij meegedacht en meegeleefd over de keuzes die ik moest maken. Naar mijn verhalen geluisterd, me altijd weer helpen verhuizen, en gezorgd voor een warm thuis. Ik zeg het niet vaak, maar dit is heel waardevol voor mij, bedankt! Mark, Frank en Anke, jullie wil ik ook bedanken voor het zijn van zulke geweldige broertjes en zusje. Ik had het niet beter kunnen treffen.

Hettie Boonman
October 2014

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1 Introduction

1.1 Motivation

On Thursday morning, 19th of January 2012, Eastman Kodak, “the 131-year-old film pioneer" filed for bankruptcy after a long struggle to adapt to an increasingly digital world. While Eastman Kodak suffered, its long-time rival Fujifilm was doing rather well. Why did these two firms fare so differently, since both saw the potential in this new emerging market. Surprisingly, Eastman Kodak was even one of the first to build a digital camera in 1975. Steve Sasson, engineer at Eastman Kodak, recalled the management’s reaction as “That is cute, but don’t tell anyone about it"[1]. Another proof of denial comes in 1988 when Sony released its first digital camera, after which an extensive research was performed by Eastman Kodak to look at the future prospective of halide film versus digital photography. What is now known as a fact was also the outcome of the study. Digital photography had the potential to replace Eastman Kodak’s established film business; however there would be no need for Eastman Kodak to hurry, because it should take roughly ten years for this new market to take off. The past has shown us that Eastman Kodak indeed dithered and did very little to prepare for this new market. Also Fujifilm observed that its revenue from film decreased from 60% of its total profit in 2000 to nearly zero in 2012. However, unlike Eastman Kodak, it was able to find new sources of revenue and survive[2].

[1] http://www.forbes.com/sites/chunkamui/2012/01/18/how-kodak-failed/
[2] http://www.economist.com/node/21542796

1.1.1 Timing of Investment Under Competition

What lesson do we learn from the Eastman Kodak-case? Timing is an important aspect in an investment decision. Waiting too long before undertaking an investment decision or changing the business-strategy, creates opportunities for competitors in the market.

In Chapter 3 and 4 of this thesis we show that it is optimal for a firm in an oligopolistic market, to invest when its expected value of waiting for the follower's position in the market is equal to its value of immediate investment. Where the value of a firm is based on expectations of future market developments. This principle is called the preemption mechanism (also known as rent equalization) and the optimal moment to invest is defined as the preemption moment. Where Fudenberg and Tirole (1985) pioneered the preemption mechanism, Smets (1991) and Smit and Ankum (1993) were the first to introduce it to the theory of real options. Within the dynamic real options setting, the incentive of the firm to invest before the optimal preemption moment is not yet strong enough. Namely, the option value to invest at some point in the future is still higher than the value of actually exercising the option. We assume that firms are symmetric ex-ante investment, and therefore they face the same investment decision. As a result, no firm shall enter yet. However, when a firm waits too long with investment, the rival will preempt by investing slightly earlier but such that its value of immediate investment is still higher than the value of waiting with investment.

Regarding the Eastman Kodak example, the firm seemed not to have learned from the history. Initially, the founder of Eastman Kodak was very well able to make a new product investment at the correct time. That is, back in the days it gave up a profitable dry-plate business to move to film, and later it invested in color film even though this was still demonstrably inferior to black and white film[3]. However, for the investment decision to move towards the digital world, Eastman Kodak was lingering too long, missed the optimal investment moment, and thereby gave the floor to its competitors who were more eager to invest.

Another example comes from the video rental industry, where Blockbuster lost in the competition game from Netflix. This happened, even though Netflix started its business only in the late nineties and Blockbuster was swelling in size

[3]http://www.forbes.com/sites/chunkamui/2012/01/18/how-kodak-failed/

since its establishment in 1985 already. However, it was Netflix who had the right timing to aggressively develop its own video-on-demand for its subscribers that allowed for streaming right to the television. Blockbuster tried to copy some moves, but it was to no avail. The firm was too late to join the video-streaming game and was, besides that, unable to add some innovations of its own. As a result, in 2010, Blockbuster had to file for bankruptcy[4].

The preemption mechanism has obtained continuous attention in academia since the early contributions of Smets (1991) and Smit and Ankum (1993). Further applications involve among others Huisman and Kort (1999) and Bouis et al. (2009). The latter stresses the importance to combine the real option approach with a multi-decision maker framework. They state that “in the western economies the extensive process of deregulation, combined with a waive of mergers and acquisitions, has resulted in an oligopolistic structure of a large number of sectors." Bouis et al. (2009) analyze oligopolistic market structures in which they find the accordion effect. That is, in case of three firms, an increase in uncertainty results in a bigger delay in the moment of third firm’s investment compared to the second firm’s moment of investment. As a result, the investment gap between the second and the third investor increases for an increase in uncertainty. The optimal moment of investment of the first entrant is more delayed compared to the second firm, therefore the investment gap between the first two investors decreases. Bouis et al. (2009) refer to these effects as the accordion effect. For very high levels of uncertainty, the first two investors shall invest simultaneously. In Chapter 4 we extend the three-firm oligopoly model of Bouis et al. (2009) by including different types of demand structures and adding capacity optimization. We find that, due to capacity optimization, the accordion effect vanishes, since it is dominated by the size effect. Namely, a higher uncertainty leads to a larger capacity choice which results in a delay of investment.

Capacity Size 1.1.2

Another aspect to take into consideration is the sizing effect, like Chapter 4 shows that taking capacity choice into account influences the accordion effect.

[4]http://money.usnews.com/money/blogs/flowchart/2010/09/23/how-netflix-and-blockbuster-killed-blockbuster

Also Dangl (1999) and Hagspiel (2011) stress the importance of capacity optimization in a real options setting. Capacity will be optimized in all subsequent chapters, where each chapter places an additional focus on another topic in the industrial organization literature. Chapter 2 considers capacity optimization in a two product market where two firms choose between a product flexible production technology and a dedicated production technology. The product flexible technology allows a firm to deviate with the production of the two products within one production line, where production with the dedicated technology is fixed. Chapter 3 and 4 show the sensitivity of the results with respect to the chosen demand structure for a duopoly and an oligopoly, respectively. Chapter 5 considers capacity optimization in an entry and exit market, where we take into account that the firm faces a time lag behind the decision to resume production where the firm prepares the production process.

Capacity Size and Different Demand Structures

Webvan Group, an e-commerce company established in the late nineties of last century, focused on selling online groceries. Its initial plan was to open operations in 26 metropolitan areas. However due to mistakes like overestimating demand and its enormous spending pattern, it had to file for bankruptcy in July 2001. Blinded by the philosophy to “get big fast", they expanded far too quickly into new cities, rather than cutting back and controlling costs in the cities where its business was already developed. The firm was loosing nearly 125 million dollar per quarter on expensive automated warehouses, computer systems, and vans to deliver the grocery products. However, since the customer growth was not rapid enough to secure a profit, this business was doomed to failure (Aspray et al. (2013)).

This is not a stand-alone example, also Boo.com[5], Pets.com[6] and eToys[7] made the same mistake to pursue with grand scale investments in a market that turned out too small for their business. These cases highlight the importance to carefully make an assumption about future demand for the considered market. In Chapter 3 and 4 we take the sensitivity of the results (e.g. capacity choice

[5] http://www.theguardian.com/technology/2005/may/16/media.business
[6] http://news.cnet.com/2100-1017-248230.html
[7] http://abcnews.go.com/Business/story?id=88548

and timing of investment) with respect to the choice of demand structure into account. We consider the additive, multiplicative and iso-elastic demand structure.

In Chapter 3 we find that when variable investment costs increase, firms invest later and in a larger capacity under additive demand, whereas for the multiplicative and iso-elastic demand functions the firms also invest later but then in less capacity. Namely, under the additive demand structure the market size increases when the firm waits for a higher demand level to invest, but the multiplicative and iso-elastic demand structures are both restricted in its market size. The multiplicative demand structure corresponds for example to a market where there is only a limited amount of customers interested in this product. Consider for instance the sale of agricultural machines, e.g. harvesters, in a region like the Netherlands where the amount of acres and farmers is limited. This results in an upper bound on demand. The additive demand structure corresponds to the majority of the markets, here there is no obvious cap on the market size. And last, the iso-elastic demand structure does pose a limitation on the market size, but not as strict as in case of the multiplicative demand structure. Reconsidering online grocer Webvan, the firm most likely assumed future demand that corresponds to the additive demand structure, since it invested in a very big capacity size under the assumption of large future growth and high variable costs. In fact, the iso-elastic demand structure could have been a better fit for their market. Namely, even though we are dealing with a growing market, Aspray et al. (2013) points out that the target group is only a small part of the population, i.e. young, female, college graduates with a household income over 70.000 dollar.

Capacity Size and Flexibility/Commitment Issues

After the second world war, a new era in manufacturing arrived, new markets were opened and firms were now competing in a global economy. However, global competition also causes large fluctuations in product demand. Where in the era before the second world war firms merely focussed on efficiency and mass production, they where now also in need of a fast response to market changes and consumer needs. The first Flexible Manufacturing System (FMS) was named System 24 and designed in 1965 by Theo Williamson for the British producer of cigarette making equipment Molins. System 24 was an automated system which was able to efficiently produce in small batches 24 hours a day, as long as production continued. However, high development costs forced the firm to close its entire machine tool division in 1973. At the same time that Molins was folding, cheap micro-technology enabled other manufacturers, like Kearney & Trecker and Cincinnaty Milacron, to develop affordable flexible manufacturing systems. Still, it were the Japanese that took the technological lead in FMS in 1977 (Forester (1989)).

An excellent and well known example comes from the automotive industry. In the fifties and the sixties of last century the North American automotive manufacturers dominated this industry. However, they relied on high-volume and inflexible plants with two, or even three, assembly lines making the same vehicle (Goyal et al. (2006)). This situation changed when Japan entered the automotive industry with FMS that allowed them to produce multiple types of cars on one single assembly line (Hagspiel (2011)). There are very few car models now whose demand is large enough to justify dedicating an entire plant to their production (Chappell (2005)). The response of the American automotive industry was to also start investing in flexibility. There is plenty of evidence that automotive manufacturers are striving for more and more flexibility. For example Tesla Motors, who began producing the Tesla S, a luxury sedan, in 2012. While the many robots in other auto factories typically perform only one function, in the new Tesla factory a robot might even do up to four: welding, riveting, bonding, and installing a component. Around the same time, also Hyundai and Beijing Motors completed a mammoth factory outside Beijing that can produce a million vehicles a year using more robots and fewer people than the big factories of their competitors and with the same flexibility as Tesla's[8].

In Chapter 2, two firms, an incumbent and an entrant, have the opportunity to optimize the level of capacity for a flexible or a dedicated production facility.

[8]http://www.nytimes.com/2012/08/19/business/new-wave-of-adept-robots-is-changing-global-industry.html?pagewanted=all&_r=0

Capacity Size and Time lags

In August 2013 , the new owner of the Saab factory in Sweden, National Electric Vehicle Sweden (NEVS), announced that they would soon start producing the 9-3 midsized car model. This model would be very similar to the model Saab stopped producing in 2011. At the moment of the announcement, the company had already recruited 300 employees. However, the start of the production was still dependent on NEVS coming to an agreement with parts suppliers, which still delayed the start of the production[9].

This example illustrates that it takes time to resume operations at a production facility. Investment and re-start decisions do not occur instantaneously. Instead, firms face a time lag after the decision to resume production, where it prepares for the production process. Namely, it takes time to find new employees and skill them to the level of the old employees before suspending the firm. However, like Bar-Ilan and Strange (1996) also mentioned, most models of irreversible investment assume that a firm can start producing immediately after the decision to (re-)start the production is made. In response to that, Chapter 5 substitutes to the existing literature that incorporates a time lag after the investment decision (e.g. Bar-Ilan and Strange (1996) and Sødal (2006)), by additionally allowing a firm to optimize the level of its capacity.

Outline of the Thesis 1.2

This thesis consists of the introduction, which is followed by four chapters. In Chapter 2, 3 and 4 capacity is optimized within a competitive environment. Chapter 5 considers the optimal investment decisions of a monopolist.

In Chapter 2 we employ a three-stage duopoly game with two products. Uncertainty is present in the sense that the firms do not know the demand realization at the moment that the investment decisions are made. In the first stage, the incumbent invests in a product flexible or a dedicated production technology. With the dedicated production technology, it will produce both products on separate production lines. The product flexible production technology gives the incumbent the opportunity to assign the available capacity freely

[9]http://europe.autonews.com/article/20130821/ANE/130829983/saab-gets-ready-to-restart-production

to either one of the products. In the second stage, the entrant decides about its optimal capacity size in its preferred production technology. Alternatively, it is also possible that, given the capacity and technology choice of the incumbent, it is optimal for the entrant to refrain from entering. After the second stage, demand is resolved and a production game will be played in the last stage of the game. The flexible production technology allows a firm to optimize production quantities within the chosen capacity size.

We find that the dedicated investment option can give the incumbent such a high 'commitment value', that it chooses the dedicated production technology over the product flexible one. This shall occur when the economic environment is not very uncertain. In this scenario, the entrant always chooses for the product flexible technology. Because the entrant optimizes its optimal production quantities given the (fixed) production quantities of the incumbent, it is also easier for the incumbent to deter the flexible entrant from the market. This result changes when a highly uncertain market environment is considered. In this situation, the entrant surprisingly chooses for the dedicated production technology. Namely, the incumbent prefers to be the only firm that can enjoy from the advantages that come with the flexible production system. As a result, it chooses its flexible capacity such high that the entrant is placed in an unpleasant production scenario. Investing dedicated is than the best strategy of the entrant, where it has the first mover's advantage in the production game. Last, we find that when firms sequentially make their investment decision, it will never occur that both firms invest in the dedicated production technology. This contrary to what Goyal and Netessine (2007) find for simultaneous investment decisions.

Chapter 3 contributes to the literature by considering the effect of three commonly used demand function, additive, multiplicative and iso-elastic demand, on the optimal investment decisions in a duopoly. In this dynamic model, uncertainty is assumed to follow a geometric Brownian motion. The firms have the opportunity to optimize the timing of investment along with the size of capacity. Under the assumption that firm roles are not predetermined, we find that the additive and iso-elastic demand structure always lead to the situation where the follower invests in a larger capacity size than the leader. However, when uncertainty and constant costs are sufficiently high, the opposite result occurs under the assumption of multiplicative demand. The use of multiplicative demand implies a cap on the total market size. This could allow the Stackelberg leader to grab the biggest share of the total market size. Furthermore, we find that an increase in variable investment costs can be explained by a direct and an indirect effect. A slightly higher variable cost decreases the optimal capacity (direct effect), but also delays investment which corresponds to a larger capacity amount (indirect effect). In case of the additive demand the indirect effect dominates, i.e. larger linear costs lead to a larger capacity choice. This is contrary to the multiplicative and iso-elastic demand function, where higher linear costs result in a later investment but smaller capacity, i.e. the direct effect dominates.

In Chapter 4 an additional sensitivity analysis of the additive and multiplicative demand structure is performed on the optimal investment decisions. This chapter considers an oligopolistic market structure with three firms. We substitute the model of Bouis et al. (2009) by considering two demand structures that allow for capacity choice, and optimize the size of capacity. Bouis et al. (2009) find the accordion effect, i.e. a higher level of uncertainty increases the third firm's threshold, decreases the second firm's threshold, and increases the first firm's threshold. For high enough uncertainty, it is optimal for the first two investors to invest simultaneously. Under capacity optimization we find that the accordion effect will not occur and is dominated by the size effect. The size effect implies that a higher level of uncertainty results in a larger capacity choice, which in turn delays investment.

Where the Chapter 2, 3 and 4 consider a competitive environment, Chapter 5 considers a monopoly. The contribution of that chapter is two-fold. It contributes to the literature of entry and exit models and to the literature of time-lag models. We consider a model with additive demand, where a firm has the opportunity to optimize the timing of investment along with the size of its capacity. After investment, the firm has infinitely many options to resume and close the operation against a fixed switching cost. Capacity optimization dominates the switching triggers of the firm. Namely, without capacity optimization, Sødal (2006) and Bar-Ilan and Strange (1996) find that a firm delays both decisions to resume and to close the production facility. Under capacity optimization, a higher level of uncertainty increases the level of capacity, which in turn increases the price intercept for which the firm makes zero profit. The optimal moment to resume and close the operation lies around this price intercept, due to the fixed switching costs. Thus, by including the capacity optimization, the firm hastens its decision to close the production facility for higher levels of uncertainty.

In the second part of Chapter 5 we assume that there exists a positive time-lag after the decision to resume the production. We find that the length of the time lag has a positive effect on the size of capacity. Bar-Ilan and Strange (1996) explain that the exit-decision truncates the downside potential of the market. Under a positive level of uncertainty, the firm expects an increase in its future revenue after the time lag. A longer time lag will only strengthen this result, i.e. the firm expects an even larger future revenue. This results in an increase of the firm's capacity choice.

2 Dedicated vs Product Flexible Production Technology: Strategic Capacity Investment Choice

This chapter is based on Boonman et al. (2014a).

Abstract

This chapter studies the optimal investment strategies of an incumbent and a potential entrant that can both choose between a product flexible technology and a dedicated technology, in a two-product market characterized by uncertain demand. The product flexible production technology has certain advantages, especially when the economic environment is uncertain. On the other hand, the dedicated production technology allows a firm to commit to production quantities. This gives strategic advantages, which can outweigh the 'value of flexibility'.

It turns out that both firms prefer, for some scenarios, the dedicated production technology. However, we find that in a game with sequential technology choices, *both* firms investing dedicated, will not be an equilibrium. Especially when the economic environment is more uncertain and one product is not very profitable, the incumbent overinvests in product flexible capacity to force the entrant to choose the dedicated technology. Then, the incumbent is the only firm with the product flexible production technology, which results in a relatively high payoff. In a similar situation but with equally profitable products, the entrant chooses the dedicated production technology because it allows the firm to commit to production quantities. This makes the entrant the first mover in the production game.

2.1 Introduction

Just two decades ago it was standard in the American and European automotive industry to install separate production lines for each vehicle type that was produced. Nowadays, most automotive manufacturers have started to invest in Flexible Manufacturing Systems (FMS) that allow production of multiple car types on a single production line. Flexible manufacturing systems were first introduced by Japanese car manufacturers that developed this new way of manufacturing when they entered the car industry. It is believed that their increased market share in the automotive market is partly due to FMS (Goyal et al. (2006)). The response of the American car industry was to also start investing in flexibility. When demand of a vehicle type drops, the firm can easily decide to shift a bigger part of the production capacity to another type of car that is produced on the same production line. This type of flexibility is in general referred to as product flexibility. The most important reason that induces manufacturers to invest in FMS is that it is a good hedge against uncertainty. In addition FMS is a way to respond to changes in competition. Goyal et al. (2006) find that “automotive manufacturers use flexibility as a ‘competitive weapon’; flexibility is deployed in market segments in which there are a larger number of flexible competitors".

However, there are many other industries in which product flexibility can evoke several efficiencies in production. Think for example about bikes or television sets. Within these industries, the manufactured products are quite similar. Therefore, it is possible to produce them on the same production line. The products in these industries are furthermore characterized by strongly fluctuating sales. In the television industry for example, innovations occur on a regular basis. Within a very short time frame the sales of a certain type of television set can drop enormously, if an improved model is introduced. Therefore, it is very desirable for a firm to be able to easily adapt the corresponding production line for the production of a different television set. However, in this chapter we show that a firm might also get value from committing to dedicated production quantities.

This chapter proposes a three stage game, where in the first stage an incumbent invests in the optimal capacity amount of either a product flexible or a dedicated production technology. The product flexible production technology allows a firm to produce both products on a single machine or production line. An entrant has the option to enter the market in the second stage. Given that the entrant invests, it will choose its optimal production technology and capacity amount. These capacity decisions are made before demand uncertainty is resolved. After the investment decision of the entrant, the market can go 'up' or 'down' with equal probability. In the final stage, the demand curve is revealed, and a production game will be played.

Research on various types of flexibility is among others surveyed in Kroll and Wasden (1990) and Karwowski and Rahimi (1990). Most of the literature on product flexibility primarily focuses on monopoly cases. Firms have to determine the optimal investment type (flexible/dedicated), the optimal (lumpy/incremental) capacity to invest in, and/or the utilization rate of the capacity. Papers that discuss the value of flexibility of a monopolist are Fine and Freud (1990), Van Mieghem (1998), Bish and Wang (2004), Chod and Rudi (2005), Tomlin and Wang (2005), and Ceryan et al. (2012). Fine and Freud (1990) consider a multi-product firm that assembles an optimal mix of flexible and dedicated capacities, where uncertainty is modeled through a revenue function with a discrete set of possible scenarios. They find that optimal capacity and expected profit is increasing in demand variance, which is consistent with our results. Bish and Wang (2004) and Chod and Rudi (2005) confirm the result of Van Mieghem (1998) that in a two-product market flexible capacity can be preferred due to financial reasons when products are perfectly positively correlated.

All these contributions consider monopolies, where the strategic effect is not taken into account when determining the choice between investing in flexible or dedicated manufacturing systems. Van Mieghem and Dada (1999) extend their monopoly model to a competition model where each firm makes decisions about capacity investment, production quantity and price. A firm is flexible in deciding which decision is postponed until after uncertainty is resolved. Patel et al. (2012) empirically investigate how some firms are able to develop more effective responses to environmental uncertainty using manufacturing flexibility. Their findings show that environmental uncertainty affects firms performance directly and indirectly through manufacturing flexibility and that operational absorptive capacity (the extend to which a firm's operational units can acquire, assimilate, and transform external information) and operational ambidexterity (pursuing both exploration and exploitation) positively moderate these mediated relationships. Considering product flexibility, Goyal and Netessine (2007) find that also under competition each firm is willing to pay more for flexibility under high demand uncertainty. For low levels of uncertainty, none of the firms will invest in flexibility, while for inbetween levels of uncertainty, the firms decide to invest in opposing production technologies. Goyal et al. (2006) explain that when there is a high demand correlation between products, the value of flexibility will be limited. Also Roller and Tombak (1990) and He et al. (2011) consider the strategic value of product flexibility. However, in those papers it is assumed that firms decide about their technology choice simultaneously. Therefore they cannot analyze the concept of 'entry deterrence'. We extend this approach by considering an incumbent-entrant situation.

Tseng (2004), Dewit and Leahy (2003) and Chang (1993) discuss flexibility in an incumbent-entrant setting. However, they do not consider product flexibility. Chang (1993) models an incumbent-entrant situation and shows that an incumbent can use product design flexibility to deter entry. The incumbent has an extra incentive to be flexible compared to a situation without a potential entrant. Contrary to Chang (1993), we consider product flexibility. We show that there are parameter combinations for which producing flexible makes it more difficult for the incumbent to deter entry in comparison to producing dedicated. This is due to the fact that a dedicated incumbent can commit to a certain production quantity.

The paper most closely related to our analysis is Anand and Girotra (2007). They consider two firms that have the opportunity to produce in a monopoly market and a competitive market. By employing early differentiation, a firm chooses the quantities for the monopoly market and the competitive market, before demand uncertainty is resolved. Delayed differentiation gives the firm the opportunity to initially produce an intermediate version of the product. Once demand uncertainty is resolved, the product will be differentiated for sale in either the monopoly market or the competitive market. They find that, when an incumbent faces a potential entrant in one of its markets, early differentiation is a better entry deterrence strategy than delayed differentiation. However, this result is found under the assumption of only one competitive market, while we assume that there are two competitive markets for the two firms. In a later section Anand and Girotra (2007) consider the issue of two competitive markets, where they find that early differentiation is, for a range of parameter values, the dominant strategy. This is however not shown under the assumption of sequential investments. Another important difference to Anand and Girotra (2007) is that, in our heterogeneous product market, the product quantity in one market influences the price of the other product. Anand and Girotra (2007) consider a different type of heterogeneity: They introduce correlation in the demand intercept, between the monopoly market and the competitive market.

Similar to Anand and Girotra (2007), we show that investing in dedicated production capacity could give the incumbent a higher (expected) profit than investing in flexible production capacity. The ability to commit oneself to production quantities gives strategic advantages. In particular, we find that the incumbent chooses the dedicated technology when demand uncertainty is low, products are equally profitable and quite substitutable. The 'value of commitment', indicating that it can give value to a firm in a competitive setting to make credible commitments, has long been advocated in the literature (Caves and Porter (1977)). This could e.g. be in the form of a contract (Rey and Salanie (1990)) or investments in a large capacity. Most important is that competitors believe that the commitment is credible and it will become very difficult to refrain from it. This gives the committed firm value and makes it able to deter possible entrants.

Besides this form of commitment, which is known in the literature, we find that in some scenarios the entrant can also benefit from being committed. In particular, when uncertainty is sufficiently low, products are equally profitable and product substitutability is low, an entrant that faces a flexible incumbent prefers investment in dedicated capacity. In such a situation, the incumbent cannot influence the entrant's production choice in the last stage, which results in a relatively high profit for the entrant. In a similar situation (i.e. low substitutability) but with a low profitability of one of the products and a more uncertain economic environment, the entrant also prefers the dedicated production choice. A higher uncertainty leads to a larger value of flexibility, i.e. high enough for the incumbent to prefer to be the only firm benefiting from the advantages of flexibility in the market. Therefore, the incumbent makes a sufficiently large capacity investment so that the entrant prefers to invest in the dedicated capacity.

Furthermore, we show that two dedicated firms cannot occur in an equilibrium, in a sequential game. This is contrary to Goyal and Netessine (2007) that make the assumption of a simultaneous technology choice. An entrant that observes a dedicated incumbent has no incentive to commit to its production quantities.

This chapter is organized as follows. The general model is presented in Section 2.2. In Section 2.3 the game is solved under appropriate assumptions. Results are discussed in Section 2.4. Section 2.5 concludes.

2.2 The Model

We consider a three stage game with two firms, an incumbent (I) and a potential entrant (E). The profit maximizing firms are assumed to be risk neutral, have full information and compete in a Cournot fashion. Demand of the two products in the market, product 1 and product 2, is uncertain. At time t=0, the incumbent has to choose between a dedicated and a flexible production technology. With the dedicated technology it has to produce each product on a separate production line. The flexible production technology allows to produce two products with a single production capacity. The incumbent makes its capacity decision at time t=0. At time t=1, the entrant determines whether it is profitable to enter the market[1]. If the entrant enters the market, it will also decide about the flexible/ dedicated production capacity at time t=1. At time t=2, demand uncertainty is resolved.

The inverse demand functions are given by:

$$P_1(q_{1,E}, q_{2,E}, q_{1,I}, q_{2,I}) = \theta - (q_{1,E} + q_{1,I}) - \gamma(q_{2,E} + q_{2,I}), \tag{2.1}$$

$$P_2(q_{1,E}, q_{2,E}, q_{1,I}, q_{2,I}) = \alpha\theta - (q_{2,E} + q_{2,I}) - \gamma(q_{1,E} + q_{1,I}), \tag{2.2}$$

where $q_{i,E}$ denotes the quantity of product i produced by the entrant and $q_{i,I}$ the quantity of product i produced by the incumbent, for $i \in \{1,2\}$. $\gamma \in (0,1)$ is the substitutability parameter. Since flexible capacity tends to be used for sub-

[1] In case we would also give the incumbent the option not to invest, this is an indication that the market is not profitable, and as a result also the entrant will not enter. Therefore this scenario is neglected.

stitutable products, we assume this parameter to be positive (Hagspiel (2011), Chod and Rudi (2005)). $\alpha \in [0, 1]$ is a measure of the profitability of product 2. Product 1 is without loss of generality assumed to be the more profitable product, except for the case of $\alpha = 1$, when product 1 and product 2 are equally profitable. The demand intercept dynamics are as follows: At time t=0, the incumbent observes demand intercept parameter θ_0. Uncertainty is present in the sense that the demand intercept θ can go 'up' or 'down' after period 1. More specifically: θ could go 'up' or 'down' by an amount equal to h, both with probability p=$\frac{1}{2}$. For tractability reasons this probability is fixed, we do not intent to make the model more involved than necessary. An upward (downward) shift of the demand intercept parameter will have a positive (negative) effect on the price, and thus on the profit. Notice that the difference in the intercept θ between an upward shift and a downward shift is equal to $2h$. Also, h denotes the uncertainty parameter (see Lemma 2.1 in Appendix 2.B.1). Equations (2.1) and (2.2) give the inverse demand functions (also known as the net-price functions). Variable production costs are denoted by parameter c. Subtracting the variable costs from gross-price functions, one obtains the net-price functions, denoted by $P_1 = p_1 - c$ for product 1 and $P_2 = p_2 - c$ for product 2, with P_1 and P_2 formulated in equation (1) and (2), respectively.

We denote the flexible capacity level by $K_{F,j}$, while $K_{1,j}$ ($K_{2,j}$) is the dedicated capacity level corresponding to product 1 (2) of firm $j \in \{I, E\}$. The costs corresponding to the investment in the flexible (dedicated) production technology are given by C_F (C_D) per unit of capacity. For the analysis in Section 2.4, we assume that $C_F = C_D$ in order to analyze the firm's technology choice, irrespective of the corresponding capacity cost. Total investment cost is $C_F K_{F,j}$ if firm $j \in \{I, E\}$ chooses the flexible production capacity, and $C_D(K_{1,j} + K_{2,j})$ if it invests in the dedicated capacity.

When the entrant enters, it incurs an additional cost equal to f. These fixed entry costs summarize potential costs arising from schooling, a new marketing plan, or licenses that need to be purchased before being able to start producing (see e.g. Tirole (1988)).

We impose the following assumption with its justifications below.

Assumption 2.1

Both firms produce up to capacity. For a dedicated firm j this means that

$q_{1,j} = K_{1,j}$ and $q_{2,j} = K_{2,j}$, with $j \in \{I, E\}$ where *I (E)* denotes the incumbent (entrant). For a flexible firm j with $j \in \{I, E\}$, we assume that $q_{1,j} + q_{2,j} = K_{F,j}$.

Assumption 2.1, often called the 'market clearance assumption', is widely used in the literature (Chod and Rudi (2005), Deneckere et al. (1997), Anand and Girotra (2007), and Goyal and Netessine (2007)). This assumption holds for instance in case of large fixed costs. In such scenarios it can be very costly to produce below the capacity level. Besides that, also knowledge will be lost. Strict labor laws prevent that employees can easily be fired, and often a high amount of money has to be paid for letting employees go. The car industry is just one example where firms often prefer to cut prices and keep production equal to full capacity, rather than underproducing (Mackintosh (2003)).

Both firms have to choose between a flexible and a dedicated production capacity. In order to find the optimal production and capacity sizes, firm j optimizes its expected future profits ($\mathbb{E}_\theta(\pi_j)$), subtracted by investment costs. For the production and capacity optimization problem considering a dedicated capacity choice of firm $j \in \{I, E\}$ is given by:

Capacity choice:

$$\Pi_j = \underset{K_{1,j} \geq 0, K_{2,j} \geq 0}{\text{Max}} \{\mathbb{E}_\theta(\pi_j) - C_D(K_{1,j} + K_{2,j}) - 1_{\{j=E\}} f\}$$

Production choice:

$$\begin{aligned}\pi_j = \underset{q_{1,j} \geq 0, q_{2,j} \geq 0}{\text{Max}} \{ & (\theta - (q_{1,E} + q_{1,I}) - \gamma(q_{2,E} + q_{2,I}))q_{1,j} \\ & + (\alpha\theta - (q_{2,E} + q_{2,I}) - \gamma(q_{1,E} + q_{1,I}))q_{2,j}\} \\ & \text{s.t. } q_{1,j} = K_{1,j}, q_{2,j} = K_{2,j}.\end{aligned}$$

The production and capacity optimization problem when firm $j \in \{I, E\}$ invests flexible is given by:

Capacity choice:

$$\Pi_j = \underset{K_{F,j} \geq 0}{\text{Max}} \{\mathbb{E}_\theta(\pi_j) - C_F K_{F,j}) - 1_{\{j=E\}} f\}$$

Production choice:

$$\pi_j = \underset{q_{1,j}\geq 0, q_{2,j}\geq 0}{\text{Max}} \{(\theta - (q_{1,E} + q_{1,I}) - \gamma(q_{2,E} + q_{2,I}))q_{1,j}$$
$$+ (\alpha\theta - (q_{2,E} + q_{2,I}) - \gamma(q_{1,E} + q_{1,I}))q_{2,j}\}$$

$$\text{s.t. } q_{1,j} + q_{2,j} = K_{F,j}.$$

Analysis of the Game-tree. 2.3

At time t=0, the incumbent has two options, investing dedicated or investing flexible. The game-tree in Figure 2.1 illustrates the choices of the incumbent and the entrant. In the following two sections, we look at both scenarios separately. Section 2.3.1 derives the optimal capacity and quantity decisions for the two firms when the incumbent invests dedicated. Section 2.3.2 solves the game-tree when the incumbent chooses for the flexible capacity.

Incumbent Invests Dedicated 2.3.1

Assume that the incumbent decides at time t=0 to invest in the dedicated production technology. At this time it also has to decide about the optimal size of capacity to invest in. One time period later, at time t=1, the entrant has the choice whether to enter with the dedicated or flexible production technology, or to stay out of the market. If the entrant enters, it will decide about the optimal capacity size at time t=1. Notice that this setup corresponds to a Stackelberg game. Since the incumbent chooses its dedicated capacity at time t=0, it fixes its production quantity before the entrant could do so. This will give the incumbent a first mover advantage. At time t=2 a production game takes place where both firms optimize their production quantities. Obviously, the production amounts of the incumbent are fixed to its dedicated capacity levels. This will also be the case for the entrant, if it chooses for the dedicated capacity. If the entrant invests in the flexible production technology, it has to determine the optimal production quantities.

Figure 2.2 shows the game tree in case the incumbent has chosen the dedicated production technology. There are eight possible outcomes for the firms,

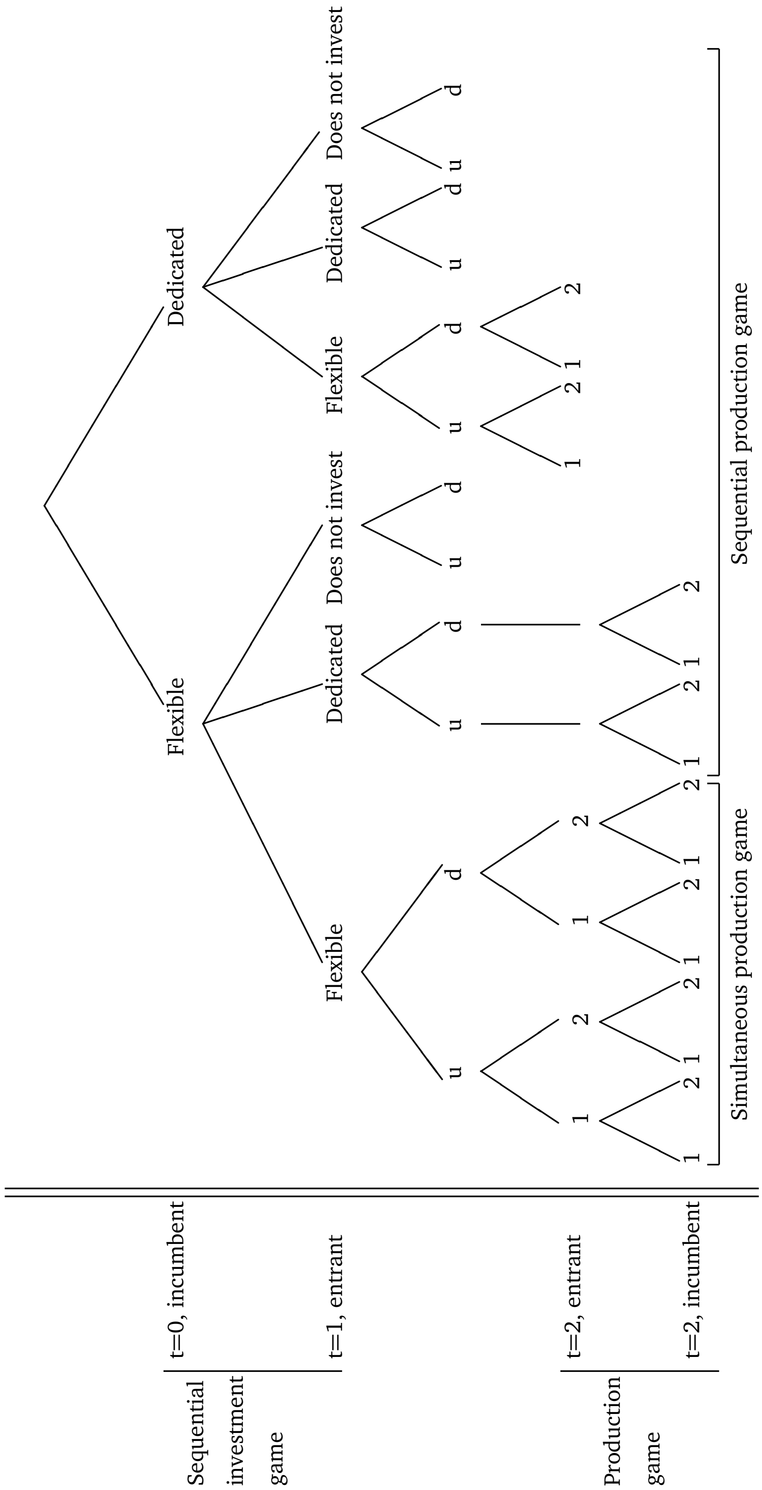


Figure 2.1: Game tree of the model.

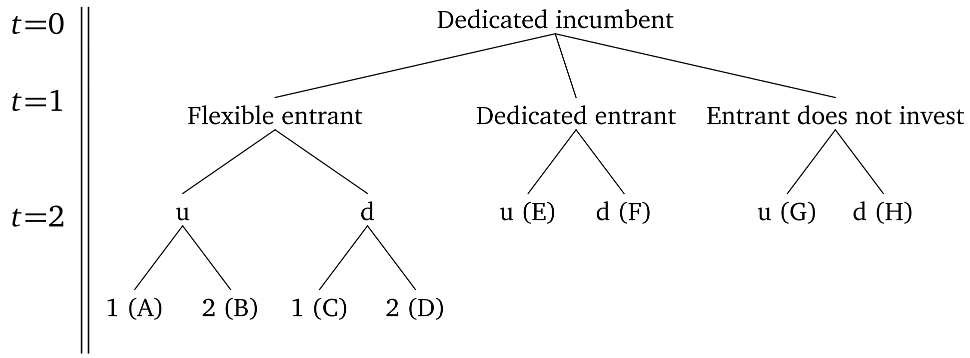


Figure 2.2: Game-tree, given that the incumbent invests dedicated.

each denoted by a letter. Six of the outcomes correspond to the situation where the entrant enters (A,B,C,D,E,F), and two correspond to the case where the entrant does not enter (G,H). After the investment decision of the entrant, the market can go 'up' or 'down', due to demand uncertainty. The entrant needs to have a strategy (i.e. production choice) for both possible outcomes ('up' or 'down'). A flexible entrant has the possibility to produce only one of the products or both products. It could for instance decide to produce only one of the products in case the market goes 'up' in period 2, and produce both products if the market goes 'down' in period 2. This strategy is labeled with AD.

In case the entrant decides to undertake investment, there are four possible capacity amounts that it could potentially invest in, depending on the strategy it would choose later. That is, it invests in capacity amount $K_{F,E,AC}$, when it uses strategy AC in combination with the product flexible capacity choice. Similarly, it would invest in capacity amount $K_{F,E,AD}$ or $K_{F,E,BD}$ if it uses strategy AD or BD, respectively, after investment in the product flexibility. The entrant invests in $(K_{1,E,EF}, K_{2,E,EF})$ if it chooses for the dedicated production technology. Notice that strategy BC is an infeasible strategy (see Lemma 1 in Appendix 2.B.2). It occurs that a flexible firm rather wants to produce both products in the down-scenario and only one product in the up-scenario. Hagspiel (2011) explains that a firm will make use of the downside potential to produce both products in order to increase total market size, when it faces low demand. If the entrant decides not to invest, it uses strategy GH.

In order to solve this game-tree, we follow the subsequent steps that specify how to derive the optimal production and capacity decision of the entrant and the optimal capacity decision of the incumbent. The game is solved backwards, since we consider an incumbent-entrant situation (see the game-tree in Figure 2.2). Therefore, we start with the optimal decisions of the entrant.

1. **Given the capacity choice of the incumbent ($K_{1,I}$ and $K_{2,I}$), we derive expressions for the optimal quantities and capacities of the entrant.** Recall that, given the dedicated technology choice of the incumbent, only the entrant has the choice between strategies; i.e. strategies AD, AC, BD, EF or GH. The entrant's profit for each possible outcome can be derived and subsequently the expected profit for each of its strategy choices. For each strategy, it will optimize its corresponding capacity and quantit(y)(ies).
2. **Identify the feasibility conditions of the entrant's strategies as a function of $K_{1,I}$ and $K_{2,I}$.** The capacity choice of the incumbent determines if a strategy from the entrant is feasible. There are two criteria for a strategy to be feasible. First, given $K_{1,I}$ and $K_{2,I}$, the entrant's endogenously determined optimal capacity and quantit(y)(ies), that correspond to this strategy, should be nonnegative. Secondly, given $K_{1,I}$ and $K_{2,I}$, a feasible strategy gives the entrant a higher profit than any of its other strategies. As a consequence, the $(K_{1,I}, K_{2,I})$-plane can be divided in five regions which only overlap on the boundaries. Each region corresponds to a feasible strategy of the entrant. To illustrate, Figure 2.3 shows the optimal feasible strategies of the entrant for each capacity combination of the incumbent firm. Notice, even though the entrant makes optimal decisions for each potential incumbent's capacity choice that it can face (technology choice, quantity and capacity optimization), it is the incumbent that determines the strategy choice of the entrant by choosing its capacities. This is due to the leader's first mover advantage.
3. **The incumbent finds the optimal $(K^*_{1,I}, K^*_{2,I})$ in all five regions.** This leads to five possible candidates for the optimal incumbent's capacity choice corresponding to each of the entrant's strategies.
4. **Given $(K^*_{1,I}, K^*_{2,I})$ for all five regions, choose the region (i.e. strategy of the entrant) that gives the incumbent the highest profit.**

The following proposition represents the first and second step for the derivation of an optimal solution. It states the optimal capacities and production quantities of the entrant. Those are derived by maximizing the respective expected profit

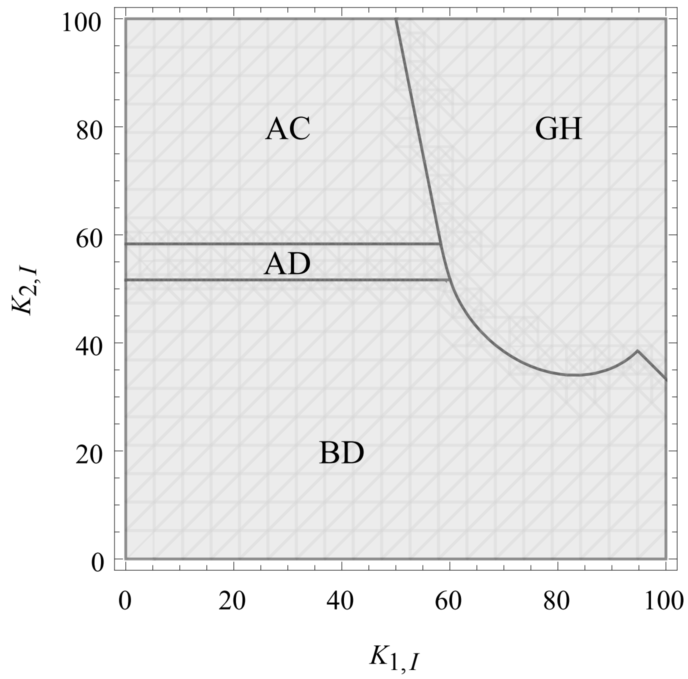


Figure 2.3: The optimal feasible regions of the five strategies of the entrant. Parameter values are γ=0.2, α =0.8, θ=100, $C_F = C_D$=10 and f=100. Notice that strategy EF, where the incumbent and the entrant choose the dedicated technology, does not have a feasible region in this figure. In Proposition 2.4 we show that this holds for all parameter regions.

functions for strategy EF, BD, AD and AC. It also formulates the conditions that should hold for each of the entrant's strategies to be feasible. (All proofs of subsequent propositions can be found in the Appendix 2.A.3.) Preferably, we would like to present concise feasibility conditions in Table 2.1. However due to messy expressions of the entrants expected optimal profit, this is impossible. Now that we know that the entrant has to choose a strategy, the notations for profit, capacity and production quantities need to be expanded. A flexible entrant that chooses strategy M, and faces market outcome N once demand is resolved obtains profit $\pi_{E,M}$, invests in capacity $K_{F,E,M}$ and produces $q_{1,E,B,BD}$ of product 1 and $q_{2,E,B,BD}$ of product 2. A dedicated entrant obtains profit $\pi_{E,M}$, and invests in capacities $K_{1,E,M}$ and $K_{2,E,M}$.

Proposition 2.1

Table 2.1 presents for each strategy the entrant's optimal capacities, optimal quantities and feasibility conditions as a function of the incumbents capacities $K_{1,I}$ and $K_{2,I}$. In case the entrant decides not to invest, i.e. strategy GH, its profit and capacities are equal to zero. This strategy is optimal when $K_{1,I}$ and $K_{2,I}$ are so high that the other strategies result in a negative expected profit for the entrant. The only set of values where the strategies can be overlapping are the boundaries between these corresponding regions.

The entrant chooses its optimal feasible strategy for each possible capacity choice of the incumbent, however it is the investment decision of the incumbent, $(K_{1,I}, K_{2,I})$, that will eventually determine which of the strategies the entrant will choose. Hence, the incumbent can 'manipulate' the strategy choice of the entrant. In the third step towards the optimal solution, the incumbent finds, within each feasible region for a strategy of the entrant, the capacity choice that gives a maximal profit. As an illustration, let us consider strategy BD, which is a possible strategy when the entrant invests in the flexible capacity. This illustration subsequently explains why we are forced to bend towards numerical analysis, the analytical expressions get so messy that it does not lead to valuable conclusions. There are two possibilities. The optimal capacity choice of the incumbent lays either within the interior of the region that belongs to strategy BD, or it results in one of the following possible boundary solutions:

1. The production quantity of product 2 of the entrant is zero.
2. The total flexible capacity of the entrant is zero.
3. The total profit of the entrant is zero.

The following proposition introduces the possible capacity choices of the incumbent considering the entrant's strategy BD.

Proposition 2.2

(i) If the optimal capacities of the incumbent lie within the interior of the region that belongs to strategy BD, the optimal capacities are given by:

$$K^*_{1,I} = \frac{\frac{1}{2}(1-\gamma\alpha)\theta}{(1-\gamma)(1+\gamma)} + \frac{\frac{1}{2}C_F - C_D}{1+\gamma}, \tag{2.3}$$

$$K^*_{2,I} = \frac{\frac{1}{2}(\alpha-\gamma)\theta}{(1-\gamma)(1+\gamma)} + \frac{\frac{1}{2}C_F - C_D}{1+\gamma}. \tag{2.4}$$

(ii) If the optimal capacities resulting from (2.3) and (2.4) lie outside the region that belong to strategy BD, the optimal capacities are equal to one of the following boundary solutions: The boundary solution for the case that the production quantity of product 2 of the entrant is zero is equal to

$$\begin{aligned} K_{1,I}^* =& 0.5\theta + \gamma K_{2,I}^* - C_D + 0.5C_F, \\ K_{2,I}^* =& \frac{(\alpha-\gamma)\theta}{1-\gamma^2} - \frac{0.5(1-\alpha)h}{1-\gamma} - \frac{C_F}{1+\gamma}. \end{aligned}$$

The boundary solution for the case that the capacity of the entrant is equal to zero is given by:

$$\begin{aligned} K_{1,I}^* =& \frac{(0.5\gamma-1,5)+\alpha(1.5\gamma-0.5)}{2(1-\gamma)}\theta - \frac{C_F}{1+\gamma}, \\ K_{2,I}^* =& \frac{(1+\alpha)\theta-2C_F}{1+\gamma} - K_{1,I}^*. \end{aligned}$$

The boundary solution for the case that the entrant's profit is equal to zero is implicitly determined by the following equation

$$\frac{d\pi_{I,BD}(K_{1,I},K_{2,I}(K_{1,I}))}{dK_{1,I}} = \frac{\partial \pi_{I,BD}}{\partial K_{1,I}} + \frac{\partial \pi_{I,BD}}{\partial K_{2,I}}\frac{\partial K_{2,I}}{\partial K_{1,I}} = 0,$$

where

$$\begin{aligned} K_{2,I}(K_{1,I}) = -(C_F+\gamma K_{1,I}-\alpha\theta) - \frac{1}{4(1-\gamma^2)}\big(&16(1-\gamma^2)^2(C_F+\gamma K_{1,I}-\alpha\theta)^2 \\ &-8(1-\gamma^2)\big(4C_F(1-\gamma)(C_F+K_{1,I}(1+\gamma)-\theta(1+\alpha)) \\ &+(1-\alpha)^2h^2(1+\gamma)+2\theta^2(1+\alpha^2-2\alpha\gamma) \\ &+(1-\gamma^2)(2K_{1,I}^2-4K_{1,I}\theta-8f)\big)\big)^{\frac{1}{2}} \end{aligned}$$

and

$$\begin{aligned} \pi_{I,BD}(K_{1,I},K_{2,I}) = \frac{1}{2}(K_{1,I}+\alpha K_{2,I})\theta - \frac{1}{2}(K_{1,I}^2+K_{2,I}^2-2\gamma K_{1,I}K_{2,I}) + \\ (K_{1,I}+K_{2,I})(\frac{1}{2}C_F - C_D). \end{aligned}$$

For the other four regions the optimal capacities of the incumbent can be derived in a similar way. In fact the optimization process of the incumbent's

Strategy BD	
$\mathbb{E}[\pi_{E,BD}] \geq 0$	$\mathbb{E}[\pi_{E,BD}] \geq \mathbb{E}[\pi_{E,\psi}] \quad \forall \psi \in \{AD, AC, EF\}$
$K_{F,E,BD} = \frac{1}{1+\gamma}\left(\frac{1}{2}(1+\alpha)\theta - C_F\right) - \frac{1}{1+\gamma}\left(-\frac{1}{2}(1+\gamma)(K_{1,I}+K_{2,I})\right)$	$q_{1,E,B,BD} = \frac{1}{2}K_{F,E,BD} - \frac{1}{4}\left(K_{1,I} - K_{2,I} - \frac{(1-\alpha)(\theta+h)}{1-\gamma}\right)$ $q_{2,E,B,BD} = \frac{1}{4}\left(K_{1,I} - K_{2,I} + 2K_{F,E,BD} - \frac{(1-\alpha)(\theta+h)}{1-\gamma}\right)$ $q_{1,E,D,BD} = \frac{1}{2}K_{F,E,BD} - \frac{1}{4}\left(K_{1,I} - K_{2,I} - \frac{(1-\alpha)(\theta-h)}{1-\gamma}\right)$ $q_{2,E,D,BD} = \frac{1}{4}\left(K_{1,I} - K_{2,I} + 2K_{F,E,BD} - \frac{(1-\alpha)(\theta-h)}{1-\gamma}\right)$

Strategy AD	
$\mathbb{E}[\pi_{E,AD}] \geq 0$	$\mathbb{E}[\pi_{E,AD}] \geq \mathbb{E}[\pi_{E,\psi}] \quad \forall \psi \in \{BD, AC, EF\}$
$K_{F,E,AD} = \frac{1}{1.5-0.5\gamma}(\frac{1}{4}(1-\alpha)h + \frac{1}{4}(3-\alpha)\theta + \frac{1}{4}(3+\gamma)K_{1,I} - \frac{1}{4}(1+3\gamma)K_{2,I} - C_F)$	$q_{1,E,A,AD} = K_{F,E,AD}$ $q_{2,E,A,AD} = 0$ $q_{1,E,D,AD} = \frac{1}{2}K_{F,E,AD} - \frac{1}{4}\left(K_{1,I} - K_{2,I} - \frac{(1-\alpha)(\theta-h)}{1-\gamma}\right)$ $q_{2,E,D,AD} = \frac{1}{4}\left(K_{1,I} - K_{2,I} + 2K_{F,E,AD} - \frac{(1-\alpha)(\theta-h)}{1-\gamma}\right)$

Strategy AC	
$\mathbb{E}[\pi_{E,AC}] \geq 0$	$\mathbb{E}[\pi_{E,AC}] \geq \mathbb{E}[\pi_{E,\psi}] \quad \forall \psi \in \{BD, AD, EF\}$
$K_{F,E,AC} = \frac{1}{2}\left(\theta - K_{1,I} - \gamma K_{2,I} - C_F\right)$	$q_{1,E,A,AC} = K_{F,E,AC}$ $q_{2,E,A,AC} = 0$ $q_{1,E,C,AC} = K_{F,E,AC}$ $q_{2,E,C,AC} = 0$

Strategy EF	
$\mathbb{E}[\pi_{E,EF}] \geq 0$	$\mathbb{E}[\pi_{E,EF}] \geq \mathbb{E}[\pi_{E,\psi}] \quad \forall \psi \in \{BD, AD, AC\}$
$K_{1,E,EF} = \frac{\theta(1-\alpha\gamma)-C_D(1-\gamma)}{2(1-\gamma^2)}$ $K_{2,E,EF} = \frac{\theta(\alpha-\gamma)-C_D(1-\gamma)}{2(1-\gamma^2)}$	$q_{1,E,E,EF} = K_{1,E,EF}$ $q_{2,E,E,EF} = K_{2,E,EF}$ $q_{1,E,F,EF} = K_{1,E,EF}$ $q_{2,E,F,EF} = K_{2,E,EF}$

Table 2.1: Feasibility conditions for each strategy of the entrant. Denote the profit of the **entrant** that uses **strategy BD** with $\pi_{E,BD}$. $K_{F,E,BD}$ denotes the optimal flexible capacity of the **entrant**, given that it produces two products if the market goes 'down' at time t=2 and two products if the market goes 'up' at time t=2 (**Strategy BD**). And $q_{1,E,B,BD}$ denotes the quantity produced if the market goes 'up' (outcome **B**) of **product 1**, for the case that the entrant chooses **strategy BD**. The other profits, capacities and quantities are defined similarly.

capacity is analytically quite involved due to the messy expressions and the many cases that arise. Therefore, we use the software program Mathematica to numerically solve the restricted optimization problem of the incumbent within each region.

Incumbent Invests Flexible 2.3.2

When the incumbent invests in the flexible production technology, the production amounts of the incumbent are not predetermined. When the entrant also invests in the flexible production technology, both firms choose simultaneously optimal production quantities at the last stage. Therefore, one has to determine a **Nash equilibrium** in this case, instead of a **Stackelberg equilibrium**.

Figure 2.4 illustrates the 16 possible outcomes that could occur when the incumbent invests in the flexible production technology (I-Y). Considering that

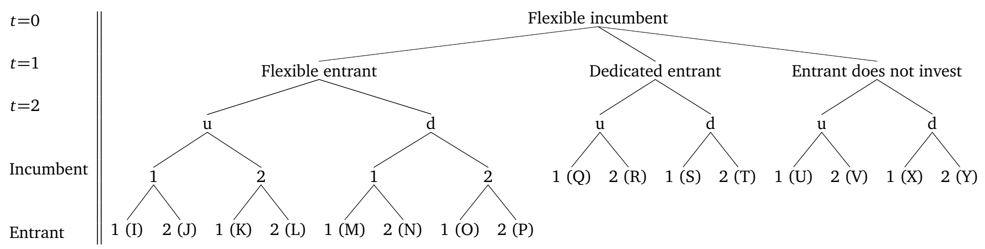


Figure 2.4: Game-tree, given that the incumbent invests flexible.

there is uncertainty about an up- or downward shift of the market, 24 possible scenarios[1] arise. If the firms choose outcome J in case the market goes 'up' and outcome N in case the market goes 'down', the combination is called scenario JN. Notice that some of these scenarios can be eliminated immediately. As was proved beforehand (see Appendix 2.B.2), a flexible firm would never produce both products in the 'up' situation and only one product in the 'down' situation. This case is always dominated by producing both products in both the 'up' and 'down' scenario. Eliminating the corresponding scenarios leads to 15 remaining scenarios given in Table 2.2.

Given each of the remaining feasible scenarios, the incumbent and the entrant simultaneously optimize their respective production quantities. Table

[1]For the case where the incumbent invests dedicated, the production decisions of the entrant in case of an up- and downwards demand shift is well defined by a *strategy*. The entrant knows for each strategy what the outcome in the market will be. In case the incumbent invests flexible however, the incumbent and the entrant decide simultaneously about the outcome in a Nash equilibrium. For each production decision of the entrant, there are several possible market outcomes, depending on the production decision of the incumbent. Therefore, for the case that the incumbent invests flexible, we introduce the notion "scenario". A *scenario* is defined to be the combination of two possible outcomes, corresponding to an up and downwards shift of the market, when demand realization is not yet resolved.

IM	IN	IO	IP	UX
JN	JP	KO	KP	UY
LP	QS	QT	RT	VY

Table 2.2: Feasible scenarios given that the incumbent invests flexible.

2.4 summarizes the equilibrium production quantities for all feasible scenarios for the incumbent and entrant. However, one should realize that even though the firms simultaneously optimize production quantities, they are not symmetric due to the incumbent-entrant setting. Hence, first we optimize for each scenario the optimal capacity size for the entrant, given the corresponding optimal production quantities. These are stated in Table 2.3. The profit of firm j, $j \in \{I, E\}$, under market outcome N after demand realization is denoted by $\pi_{j,N}$. The capacity choice of flexible firm j that employs scenario M is denoted by $K_{F,j,M}$, and of a dedicated entrant by $K_{1,E,M}$ and $K_{2,E,M}$. The production quantities of flexible firm j are denoted by $q_{1,j,N,M}$ for product 1 and $q_{2,j,N,M}$ for product 2.

In order to solve the production game, the following game in normal-form has to be solved in the 'up' and 'down' situation respectively:

Up:

$$\begin{array}{ll} & \text{Decision Entrant} \\ & \begin{array}{cccc} F1 & F2 & D & 0 \end{array} \\ \begin{array}{lc} \text{Decision} & F1 \\ \text{Incumbent} & F2 \end{array} & \begin{pmatrix} (\pi_{I,I}, \pi_{E,I}) & (\pi_{I,J}, \pi_{E,J}) & (\pi_{I,Q}, \pi_{E,Q}) & (\pi_{I,U}, \pi_{E,U}) \\ (\pi_{I,K}, \pi_{E,K}) & (\pi_{I,L}, \pi_{E,L}) & (\pi_{I,R}, \pi_{E,R}) & (\pi_{I,V}, \pi_{E,V}) \end{pmatrix} \end{array}$$

Down:

$$\begin{array}{ll} & \text{Decision Entrant} \\ & \begin{array}{cccc} F1 & F2 & D & 0 \end{array} \\ \begin{array}{lc} \text{Decision} & F1 \\ \text{Incumbent} & F2 \end{array} & \begin{pmatrix} (\pi_{I,M}, \pi_{E,M}) & (\pi_{I,N}, \pi_{E,N}) & (\pi_{I,S}, \pi_{E,S}) & (\pi_{I,X}, \pi_{E,X}) \\ (\pi_{I,O}, \pi_{E,O}) & (\pi_{I,P}, \pi_{E,P}) & (\pi_{I,T}, \pi_{E,T}) & (\pi_{I,Y}, \pi_{E,Y}) \end{pmatrix} \end{array}$$

In the production game, the incumbent decides whether it produces only one (F1) or both (F2) products. The entrant has four options: produce only one (F1) or both (F2) products with the flexible capacity, produce up to the dedicated capacity (D), or stay out of the market and produce nothing (0). No firm should have the incentive to deviate from the occurring outcome. Therefore, all Nash equilibria of this game have to be determined. The equilibrium where the market ends up in, will satisfy that both the optimal capacities of the firms, as well as the equilibrium quantities of the second product, have to be nonnegative (Table 2.3 and 2.4). Proposition 2.3 summarizes the feasibility conditions for each strategy that have to be satisfied to be a feasible Nash equilibrium.

Proposition 2.3
The equilibrium production capacities of the entrant are stated in Table 2.3. Table 2.4 gives the unique equilibrium quantities of product 2 for both firms. For the case that the incumbent invests in product flexibility, the feasibility conditions for scenario JN is stated in Table 2.5. Feasibility conditions for the other scenarios can be obtained similarly.

Scenario	Optimal flexible capacity of entrant $K_{F,E}$	
IM	$\frac{1}{2}(\theta - K_{F,I,IM} - C_F)$	
IN	$\frac{1}{6+2\gamma}\left((3+\alpha)\theta + (1-\alpha)h - (3+\gamma)K_{F,I,IN} - 4C_F\right)$	
IO	$\frac{1}{1.75+0.25\gamma}\left(\frac{1}{8}(7+\alpha)\theta + \frac{1}{8}(1-\alpha)h - \frac{1}{4}(\gamma+3)K_{F,I,IO} - C_F\right)$	
IP	$\frac{1}{6+2\gamma}\left((3+\alpha)\theta + (1-\alpha)h - (3+\gamma)K_{F,I,IP} - 4C_F\right)$	
JN	$\frac{1}{1+\gamma}\left(\frac{1}{2}(1+\alpha)\theta - \frac{1}{2}(1+\gamma)K_{F,I,JN} - C_F\right)$	
JP	$\frac{1}{1+\gamma}\left(\frac{1}{2}(1+\alpha)\theta - \frac{1}{2}(1+\gamma)K_{F,I,JP} - C_F\right)$	
KO	$\frac{1}{6+2\gamma}\left((3+\alpha)\theta - 2(1+\gamma)K_{F,I,KO} - 4C_F\right)$	
KP	$\frac{1}{1.25+0.75\gamma}\left((0.625+0.375\alpha)\theta + \frac{1}{8}(1-\alpha)h - \frac{1}{2}(1+\gamma)K_{F,I,KP} - C_F\right)$	
LP	$\frac{1}{1+\gamma}\left(\frac{1}{2}(1+\alpha)\theta - \frac{1}{2}(1+\gamma)K_{F,I,LP} - C_F\right)$	
	Optimal dedicated capacities of entrant ($K_{1,E}, K_{2,E}$)	
RT	$\frac{2(1-\alpha\gamma)\theta-(1-\gamma)(2C_D+(1+\gamma)K_{F,I,RT})}{4(1-\gamma^2)}$	$\frac{2(\alpha-\gamma)\theta-(1-\gamma)(2C_D+(1+\gamma)K_{F,I,RT})}{4(1-\gamma^2)}$
QT	$\frac{\frac{3}{2}(1-\alpha\gamma)\theta+\frac{1}{4}(1-\alpha)(1+\gamma)h-\frac{5}{4}(1-\gamma^2)K_{F,I,QT}-\frac{3}{2}(1-\gamma)C_D}{3(1-\gamma^2)}$	$\frac{\frac{3}{2}(\alpha-\gamma)\theta-\frac{1}{8}(1-\alpha)(1+\gamma)h-\frac{1}{8}(1-\gamma^2)K_{F,I,QT}-\frac{3}{2}(1-\gamma)C_D}{3(1-\gamma^2)}$
QS	$\frac{(1-\alpha\gamma)\theta-(1-\gamma^2)K_{F,I,QS}-(1-\gamma)C_D}{2(1-\gamma^2)}$	$\frac{(\alpha-\gamma)\theta-(1-\gamma)C_D}{2(1-\gamma^2)}$

Table 2.3: Optimal capacities of the entrant for all feasible scenarios.

Scenario M	Outcome N	Quantity product 2 incumbent $q_{2,I,N,M}$	Quantity product 2 entrant $q_{2,E,N,M}$
IM	I	0	0
	M	0	0
IN	I	0	0
	N	0	$\frac{2(\gamma-\alpha)\theta}{\frac{1}{2}(\gamma+3)(\gamma-1)}+\frac{\frac{1}{2}(1-\alpha)h-2C_F}{1.5+\frac{1}{2}\gamma}$
IO	I	0	0
	O	$\frac{\frac{1}{8}(1-\alpha)(1-\gamma)h+((\frac{3}{8}-\frac{9}{8}\gamma)+\alpha(1\frac{7}{8}+\frac{1}{8}\gamma))\theta}{(\gamma+7)(1-\gamma)}+\frac{\frac{1}{4}(\gamma+11)K_{F,I,GM}-C_F}{\gamma+7}$	0
IP	I	0	0
	P	$\frac{K_{F,I,IP}}{2}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$	$\frac{\frac{1}{4}(3+\alpha)\theta-\frac{1}{4}(3+\gamma)K_{F,I,IP}+\frac{1}{4}(1-\alpha)h-C_F}{3+\gamma}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$
JN	J	0	$\frac{1}{4}\left(\frac{2(\alpha-\gamma)\theta}{1-\gamma^2}-\frac{2C_F}{1+\gamma}\right)$
	N	0	$\frac{1}{4}\left(\frac{2(\alpha-\gamma)\theta}{1-\gamma^2}-\frac{2C_F}{1+\gamma}\right)$
JP	J	0	$\frac{1}{4}\left(\frac{2(\gamma-\alpha)\theta}{1-\gamma^2}-\frac{2C_F}{1+\gamma}\right)$
	P	$\frac{K_{F,I,JP}}{2}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$	$\frac{\frac{1}{4}(1+\alpha)\theta-\frac{1}{2}(1+\gamma)K_{F,I,JP}-C_F}{2(1+\gamma)}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$
KO	K	$\frac{1}{4}\left(2K_{F,I,KO}-\frac{(1-\alpha)\theta}{1-\gamma}+\frac{(\frac{3}{4}+\frac{1}{4}\alpha)\theta-\frac{1}{2}(1+\gamma)K_{F,I,KO}-C_F}{1.5+0.5\gamma}\right)$	0
	O	$\frac{1}{4}\left(2K_{F,I,KO}-\frac{(1-\alpha)\theta}{1-\gamma}+\frac{(\frac{3}{4}+\frac{1}{4}\alpha)\theta-\frac{1}{2}(1+\gamma)K_{F,I,KO}-C_F}{1.5+0.5\gamma}\right)$	0
KP	K	$\frac{\frac{5}{8}\theta+\frac{1}{8}(1-\alpha)h-\frac{1}{2}(1+\gamma)K_{F,I,KP}-C_F}{5+3\gamma}-\frac{\frac{1}{8}(2-\alpha)\theta}{1-\gamma}+\frac{1}{2}K_{F,I,KP}$	$\frac{\frac{5}{8}\theta+\frac{1}{8}(1-\alpha)h-\frac{1}{2}(1+\gamma)K_{F,I,KP}-C_F}{2.5+1.5\gamma}+\frac{(0.791667-0.125\gamma)\alpha-(1.25-0.75\gamma)}{(1-\gamma)(1.25+0.75\gamma)}\theta$
	P	$\frac{K_{F,I,KP}}{2}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$	0
LP	L	$\frac{K_{F,I,LP}}{2}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$	$\frac{\frac{1}{2}(1+\alpha)\theta-\frac{1}{2}(1+\gamma)K_{F,I,LP}-C_F}{2(1+\gamma)}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$
	P	$\frac{K_{F,I,LP}}{2}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$	$\frac{\frac{1}{2}(1+\alpha)\theta-\frac{1}{2}(1+\gamma)K_{F,I,LP}-C_F}{2(1+\gamma)}-\frac{(1-\alpha)\theta}{6(1-\gamma)}$
RT	R	$\frac{K_{F,I,RT}}{2}-\frac{(1-\gamma)(\theta+h)}{8(1-\gamma)}$	$K_{2,E,RT}$
	T	$\frac{K_{F,I,RT}}{2}-\frac{(1-\gamma)(\theta-h)}{8(1-\gamma)}$	$K_{2,E,RT}$
QT	Q	0	$K_{2,E,QT}$
	T	$\frac{3K_{F,I,QT}}{8}-\frac{(1-\gamma)(\theta-h)}{8(1-\gamma)}$	$K_{2,E,QT}$
QS	Q	0	$K_{2,E,QS}$
	S	0	$K_{2,E,QS}$
UX	U	0	0
	X	0	0
UY	U	0	0
	Y	$\frac{K_{F,I,UY}}{2}-\frac{(1-\alpha)\theta}{4(1-\gamma)}$	0
VY	V	$\frac{K_{F,I,VY}}{2}-\frac{(1-\alpha)\theta}{4(1-\gamma)}$	0
	Y	$\frac{K_{F,I,VY}}{2}-\frac{(1-\alpha)\theta}{4(1-\gamma)}$	0

Table 2.4: Optimal quantities for product 2 of the incumbent and the entrant for all feasible scenarios. The optimal quantity of product 1, if a firm does not produce product 2, is equal to the total capacity of this firm. If the optimal quantity of product 2 is strictly positive, the quantity of product 1 is simply the total capacity of the firm minus the production quantity of product 2 in that scenario.

<table>
<tr><th colspan="2">Strategy JN</th></tr>
<tr><td colspan="2">$\mathbb{E}[\pi_{I,JN}] \geq \mathbb{E}[\pi_{I,\varphi}] \quad \forall \varphi \in \{JP, LP\} \qquad \mathbb{E}[\pi_{I,JN}] \geq 0$
$\mathbb{E}[\pi_{E,JN}] \geq \mathbb{E}[\pi_{E,\phi}] \quad \forall \phi \in \{IN, QS, UX, IM\}$</td></tr>
<tr><td>$K_{F,E,JN} = \frac{\frac{1}{2}(1+\gamma)\theta - \frac{1}{2}(1+\gamma)K_{F,I,JN} - C_F}{1+\gamma}$

$q_{1,I,J,JN} = K_{F,I,JN}$
$q_{1,I,N,JN} = K_{F,I,JN}$</td><td>$q_{1,E,J,JN} = \frac{1}{2}\left(\frac{\theta(1-\alpha\gamma)}{(1-\gamma^2)} + \frac{h(1-\alpha)}{2(1-\gamma)} - \frac{C_F}{(1+\gamma)} - K_{F,I,JN}\right)$
$q_{1,E,N,JN} = \frac{1}{2}\left(\frac{\theta(1-\alpha\gamma)}{(1-\gamma^2)} - \frac{h(1-\alpha)}{2(1-\gamma)} - \frac{C_F}{(1+\gamma)} - K_{F,I,JN}\right)$
$q_{2,E,J,JN} = \frac{1}{4}\left(\frac{2(\alpha-\gamma)\theta}{1-\gamma^2} - \frac{2C_F}{1+\gamma}\right)$
$q_{2,E,N,JN} = \frac{1}{4}\left(\frac{2(\alpha-\gamma)\theta}{1-\gamma^2} - \frac{2C_F}{1+\gamma}\right)$</td></tr>
</table>

Table 2.5: Feasibility conditions for strategy JN. The feasibility conditions for the other strategies can analogously be obtained.

Table 2.3 shows that the optimal capacities that correspond to scenario IN and IP ($K_{F,E,IN}$ and $F_{F,E,IP}$) are the same. Under strategy IN and IP, both firms invest in the flexible production. However, in case the market goes down, the incumbent firm produces only product 1 under scenario IN while it produces both products under scenario IP. The optimal production quantities of the incumbent are not affected by the capacity choice of the entrant ($K_{F,E,IN}$ or $K_{F,E,IP}$). These optimal incumbents optimal production quantities are inserted in the entrants expected future profit function, which it maximizes with respect to capacity choice. As a result, the capacity size of the entrant is also not affected by the choice of the incumbent to produce either one or two products. For the same reason we also find that the optimal capacities, corresponding to scenario JN, JP and LP, are equal.

The incumbent first chooses its capacity. Hence, the incumbent again enjoys the first mover advantage with respect to the capacity choice (note that the production game is played simultaneously). As in Section 2.3.1, each scenario gives the incumbent many different boundary and interior solutions, of which it chooses the optimal one. Besides that, also expected profit expressions are messy, which makes an analytical solution for the incumbents optimal capacity choice impossible. We solve this using the software program Mathematica.

2.4 Results

2.4.1 The Value of Flexibility Versus the Value of Commitment

Earlier literature shows that increasing demand uncertainty substantially raises the value of flexible production systems. This holds for several types of flexibility (Hagspiel (2011), Chod and Rudi (2005), and Anupindi and Jiang (2008), Yang et al. (2011)). In this chapter we consider product flexibility. The advantage of product flexibility is that it enables a firm to shift production around within one production line. This gives value to a firm, i.e. the value of flexibility. On the other hand, dedicated production systems generate a value of commitment. Combined with the market clearance assumption, an incumbent with a dedicated production system commits to the production amounts it has chosen initially when deciding about the capacity level. The entrant is aware of this inflexibility and adapts to the production choice of the incumbent. Therefore, the dedicated incumbent will enjoy the Stackelberg first mover advantage (Anand and Girotra (2007)). Besides this form of commitment value, we find a new type of commitment, namely commitment on the side of the entrant. This happens when the incumbent invests in a flexible capacity and the entrant in a dedicated capacity. Here, the incumbent cannot influence the quantity decision of the entrant in the production game, which gives additional value to the entrant.

There are two ways in which a firm can produce, either a firm invests dedicated or a firm invests flexible. This leads to two possible objective values. Comparing these objective values will make clear what dominates, the value of flexibility or the value of being committed to produce one specific technology. However, we cannot present a consistent formula for the value of flexibility and the value of commitment, since the tradeoff between these values is discussed differently in different subsections of the results.

Figure 2.5 illustrates the tradeoff between the two explained types of commitment value and the value of flexibility. Notice that the substitutability parameter is chosen to be low (γ=0.2). When the substitutability parameter is high, the products are so similar that both firms want to produce just the more profitable product 1 for all considered market situations. This holds for both the dedicated and the flexible production technology. However, if the substitutability parameter is low, it might be optimal to produce both products.

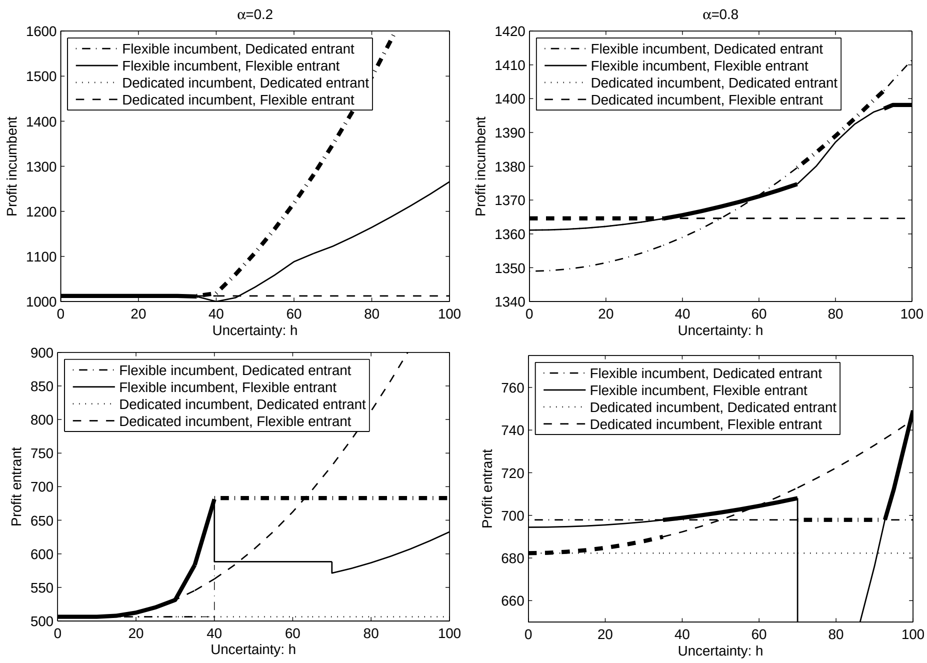


Figure 2.5: The profit of the incumbent (upper graph) and the entrant (lower graph) for the four possible occurring market situations. Parameter values are γ=0.2, θ=100, $C_F = C_D$=10 and f=0. Not all lines are visible in the figures. This indicates that some market outcomes result in the same profit. For the two upper graphs, the profit of a dedicated incumbent that faces a flexible entrant overlays the profit when it faces a dedicated entrant, for all levels of uncertainty. The thick lines indicate the profit corresponding to the resulting technology equilibrium.

The two left graphs in Figure 2.5 illustrate the case where profitability of product 2 is relatively low (α=0.2). In combination with a low product substitutability (γ=0.2), Hagspiel (2011) has shown that in such a case the *value of flexibility* is larger. Therefore, the incumbent will in equilibrium invest in the flexible production technology. The lower left graph of Figure 2.5 illustrates that, given the flexible technology choice of the incumbent, the entrant's equilibrium investment choice depends on the level of uncertainty. For low levels of uncertainty the entrant prefers to be in a symmetric market, and will also invest in the flexible production technology. However, for high levels of uncertainty both firms obtain a higher profit when the entrant invests in the dedicated production technology. Here, the incumbent prefers to be the only firm with the flexible capacity in the market and makes a sufficiently high capacity investment, which forces a flexible entrant to make a production choice that is less profitable than when it would have chosen to be dedicated. The downward jump in the entrant's profit function and the kink in the incumbent's profit function, when both firms invest in the flexible capacity, is caused by the fact that for lower levels of uncertainty the entrant is the only producer of product 2, and the incumbent after the kink (see left hand plots around h =40).

The right graphs of Figure 2.5 illustrate the case when the profitability of product 2 is high (α=0.8). These graphs illustrate where the value of commitment dominates the value of flexibility for the incumbent, as well as for the entrant. The region where the *incumbent's value of commitment* dominates its value of flexibility, is seen in the upper right graph. It shows that investing dedicated is more profitable for the incumbent for a less uncertain environment. A dedicated incumbent can fix its capacity at time t=0. The entrant has to adapt afterwards to these production quantities. This gives the incumbent so much value that it prefers to invest dedicated over flexible (despite all its well known advantages). However, in case of a very uncertain market environment, the value of flexibility will dominate, and the incumbent will prefer the flexible production technology.

Result 2.1
Under some market conditions (economic environment not very uncertain, similar profitability of both products, low substitutability) it is more profitable for the incumbent to invest in the dedicated production technology.

The region where the *entrant's value of commitment* dominates its value of flexibility is shown in the lower right graph of Figure 2.5. When the incumbent invests in the flexible production technology and an intermediate demand uncertainty is considered, the entrant obtains a larger profit from investment in the dedicated capacity than from investment in the flexible capacity. Here, it is less profitable for the entrant to also invest in the flexible capacity, because a flexible incumbent can influence the optimal quantity decision of the flexible entrant in the production game, which lowers the entrant's optimal quantity and therefore its profit. Therefore, the entrant fixes production to its dedicated capacities, before uncertainty is resolved. However, when the demand uncertainty is somewhat higher, the value of flexibility is so high that also the entrant obtains a higher profit from investment in the flexible production capacity.

Result 2.2
When the incumbent has invested in the flexible capacity, it is for some market conditions (intermediate levels of uncertainty, similar profitability of both products, low substitutability) more profitable for the entrant to invest in the dedicated production technology.

Equilibria 2.4.2

Where our incumbent-entrant setup results in sequential capacity choice, Goyal and Netessine (2007) consider a simultaneous capacity game. They show that when the flexible and dedicated capacity are equally expensive, an economic environment with low uncertainty results in a market where either both firms invest in the dedicated capacity, or one firm invests dedicated and the other firm flexible. For inbetween levels of uncertainty, the two firms choose to be symmetric and invest in the same production technology. For high values of uncertainty, the firms choose in equilibrium to both invest in the flexible production technology, or one firm invests flexible and the other firm invests dedicated.

Contrary to Goyal and Netessine (2007), we can show that a sequential game will never lead to an equilibrium where both firms invest in the dedicated capacity. This we see in the two considered market situations in Figure 2.5. Proposition 2.4 states that this result holds for all market conditions.

Proposition 2.4
In a market with sequential investment, both firms investing in the dedicated production technology will not be an equilibrium.

Entry Accommodation and Entry Deterrence 2.4.3

The incumbent can react in several ways to an entry threat. In particular, we can distinguish between entry accommodation, entry deterrence, and blockaded entry (see e.g. Tirole (1988)). With blockaded entry, the optimal monopoly capacities of the incumbent already cause that the entrant will not enter the market. Entry deterrence also prevents the entrant to enter the market. To achieve this however, the incumbent needs to overinvest (invest in more capacity than a monopolist would do) in order to prevent entry. Entry accommodation will occur when the investment that is necessary to prevent entry is too large, i.e. it is too costly, so that it is better for the incumbent to take entry of the competitor for granted.

Entry deterrence and blockaded entry have as consequence that the entrant will not enter the market. Therefore, those strategies correspond for a dedicated incumbent to 'strategy GH', and for a flexible incumbent to 'scenarios UX, UY or VY'.

We now derive the minimal level of fixed entry cost, denoted by f^*, that leads to entry deterrence. Tables 2.6 and 2.7 illustrate the results for an uncertainty level h =20, in case of a dedicated and a flexible incumbent, respectively. Note that in the situation of low demand uncertainty it is more difficult for an incumbent to deter a flexible entrant than a dedicated entrant (as is also illustrated in Figure 2.5). For some levels of entry cost f, the incumbent manages to deter the dedicated entrant, but not the flexible entrant. Therefore, the incumbent faces a flexible entrant for a fixed cost of entry slightly smaller than f^*.

$\alpha \downarrow / \gamma \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	43.5	43.4	43.4	43.4	43.4	43.4	43.4	43.4	43.4
0.1	50.15	43.4	43.4	43.4	43.4	43.4	43.4	43.4	43.4
0.2	56.8	49.1	43.4	43.4	43.4	43.4	43.4	43.4	43.4
0.3	60.3	55.1	48.2	43.4	43.4	43.4	43.4	43.4	43.4
0.4	62.3	58.25	53.45	47.3	43.4	43.4	43.4	43.4	43.4
0.5	62.25	59.6	56.25	51.95	46.5	43.4	43.4	43.4	43.4
0.6	61.4	59.1	57.1	54.35	50.6	45.8	43.4	43.4	43.4
0.7	62.5	58.95	56.34	54.45	52.5	49.4	45.15	43.4	43.4
0.8	65.8	61	57.2	54.1	51.9	50.3	48.2	44.5	43.4
0.9	71.28	65.55	60.7	56.55	53.2	50.3	48.2	46.6	43.95
1	78.9	72.35	66.8	62	57.9	54.3	51.1	48.3	45.7

Table 2.6: Minimal level of fixed entry cost, f^*, for which the dedicated incumbent is able to deter entry. Parameter values are θ=100, h =20, and $C_D = C_F$=10.

$\alpha\downarrow/\gamma\rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	125.8	93.5	61.05	43.4	43.4	43.4	43.4	43.4	43.4
0.1	137.6	110	80.1	49.45	43.4	43.4	43.4	43.4	43.4
0.2	142.1	120.9	96.1	68.1	43.4	43.4	43.4	43.4	43.4
0.3	138	124.6	106.4	83.8	57.1	43.4	43.4	43.4	43.4
0.4	133.8	118.6	109.3	93.7	72.9	46.8	43.4	43.4	43.4
0.5	118.7	115	107.1	95.7	82.5	63	43.4	43.4	43.4
0.6	113.4	103	97.2	92.7	83.7	72.6	53.9	43.4	43.4
0.7	101.6	97.9	89.7	82.6	79.6	72.8	64	44.7	43.4
0.8	86.8	83.4	79.9	77.1	71.7	65.6	63.9	56.3	43.4
0.9	76	70.8	67.7	64.5	62.3	60.1	57.7	52	49.5
1	78.9	72.35	66.8	62	57.9	54.3	51.1	48.3	45.7

Table 2.7: Minimal level of fixed entry cost, f^*, for which the flexible incumbent is able to deter entry. Parameter values are θ=100, h =20, and $C_D = C_F$=10.

Tirole (1988) has shown already that an incumbent is unable to deter entry when, upon entry, the entrant does not need to incur any fixed entry costs. We confirm this result. In the presence of positive entry costs, however, we find the following result stated in Result 2.3. It is easily shown mathematically that a lower f^* (minimum entry cost to deter the entrant) corresponds to a lower over-investment level needed to deter entry for the incumbent.

Result 2.3
When demand uncertainty is relatively low, a dedicated incumbent overinvests less than or equal to that of a flexible incumbent, in order to deter the flexible entrant.

If the incumbent invests *dedicated*, it chooses both the production quantities and the capacities in the initial period t=0, while the flexible entrant determines production quantities once demand realization is resolved. Hence, the incumbent makes its production decision before the entrant does so. The production decisions are made *sequentially*, and therefore one has to derive a *Stackelberg equilibrium*, with the incumbent as leader. On the other hand, if the incumbent invests *flexible*, it can make its production decision once demand uncertainty is resolved (i.e. after its capacity decision). Then the two firms decide *simultaneously*, in the last stage of the game, about the production quantities. Consequently, a *Nash equilibrium* has to be derived. Therefore, the dedicated incumbent can deter entry for a lower minimal level of fixed entry cost.

If the incumbent chooses for the dedicated capacity, the value of commitment is easily understood for the case of $\alpha = 0$, i.e. the second product it not profitable. Assume that substitutability is low. Table 2.7 shows that in that case it is very difficult for a flexible incumbent to deter entry. Table 2.6 however shows that for these parameter values a dedicated incumbent can deter entry relatively easy, i.e. entry deterrence is already realized for low values of the fixed entry cost. Investing *dedicated* makes the incumbent stronger, in the sense that it can restrict to produce just product 1 in the first stage already. The entrant knows this and has to adapt its production quantities to the fixed quantities of the incumbent. With the *flexible* capacity, the incumbent places itself in a weaker position since it allows the entrant to have as much influence on the (Nash) equilibrium as itself. When the capacity of a flexible incumbent is very high, it might be more profitable for a flexible firm to give some of product 2 away for free, than to produce too much of product 1. (Recall that the market clearance assumption forces a firm to produce up to capacity.) A flexible incumbent thus produces a positive amount of a non-profitable product, while a dedicated incumbent will only produce the profitable product. Therefore, a dedicated incumbent can deter entry already for lower fixed entry cost.

The results presented in Tables 2.6 and 2.7 give us further insights: In the upper right triangle, the same fixed entry cost, i.e. f^*=43.4, is needed to prevent entry. The combination of low profitability of product 2 and high substitutability causes that only two strategies are feasible for the entrant: to produce only product 1 in the two possible market scenarios, or do not invest at all. Producing product 2 is never feasible for the dedicated entrant. Therefore, small changes in substitutability, profitability and uncertainty will not change the profit of the entrant here. Furthermore, there is no difference between a dedicated incumbent and a flexible incumbent for this parameter ranges. Consequently, for all situations where the entrant will produce only product 1 or invest not at all, the minimum fixed entry cost where deterrence will occur stays the same.

Second, the effect of profitability of product 2 on f^* is non monotonic. Consider the case that the incumbent invests in the dedicated production technology (see Table 2.6). Here, the entrant is the only firm that will enjoy the advantages of shifting production of the two products within the flexible capacity. For a very low value of α, the entrant only produces the more profitable product in both market scenarios. For a somewhat higher value of α, also product 2 becomes more profitable. In this case, the entrant produces product 1 when the market goes 'up', and both products when the market goes 'down'. That is, the flexible capacity becomes valuable for the entrant and a higher fixed entry cost f^* is needed to deter the entrant. For higher values of α, the profitability of product 2 is such high that the firms always produce both products. In this case, the entrant cannot not use the advantage of the flexible production technology. This results in a lower fixed entry cost is needed to deter the entrant. For very high values of α, the profitability of the second product is such high that the entrant can obtain a high profit by entering the market and producing two products. For those values of α, a higher fixed entry cost is needed to deter the entrant.

Next, we consider the case where the incumbent invests in the flexible production technology (see Table 2.7). The incumbent has the first mover advantage in the capacity game and is able to enjoy the advantages of the flexible capacity even more than the entrant. In the case that the profitability of product 2 is very low, the incumbent produces only product 1, but allows the entrant to shift the production of two products within the flexible capacity. Hence, a higher fixed entry cost f^*, is needed to deter the entrant. For a higher level of the profitability of product 2, the flexible capacity becomes more valuable and the incumbent chooses its capacity such that it is optimal to produce 1 product when the market goes 'up' and both products when the market goes 'down'. Under this capacity choice, it is optimal for the entrant to produce both products in the two market scenarios, and it is not able to use the advantage of its flexible technology. Therefore, the dedicated incumbent can deter entry for a lower minimal level of fixed entry cost.

Third, comparing the bottom rows of Tables 2.6 and 2.7 shows that it does not matter for the incumbent whether it invests flexible or dedicated when both products are equally profitable ($\alpha = 1$). The minimum fixed entry cost, f^*, that is necessary for the incumbent to be able to deter entry is the same in this case. The reason is that the firms have no incentive to produce one product over the other, if both products are equally profitable. The firms will always produce both products and assign half of their capacity to each product. Therefore, it is optimal to keep this assignment fixed, even if a firm invests flexible.

2.4.4 Entry Deterrence Facing a Highly Uncertain Economic Environment

Proposition 2.4 shows that it is never optimal for an entrant to invest in the dedicated production technology when it faces a dedicated incumbent. Therefore, when we create a similar table as Table 2.6 for a high level of uncertainty (h =80), at f^* the incumbent faces a flexible entrant. However, Figure 2.5 shows that a flexible incumbent can face a dedicated entrant for a high enough level of uncertainty. Table 2.8 and 2.9 illustrate the minimal level f of fixed entry cost that leads to entry deterrence in case of a dedicated and flexible incumbent, respectively, for a high level of uncertainty (h =80).

$\alpha \downarrow / \gamma \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	312.8	272.6	228.9	182	132.7	87.9	54.5	43.4	43.4
0.1	301.4	269.8	233.1	192.1	146.7	99.7	61.7	43.4	43.4
0.2	280.8	258.7	231.0	197.4	158.0	112.9	70.7	44.7	43.4
0.3	251.5	238.4	220.1	195.7	164.4	125.7	81.5	48.7	43.4
0.4	219.4	210.1	200.6	185.6	164.0	133.9	94.1	55.3	43.4
0.5	190.9	181.2	173.4	166.6	154.7	135.1	105.1	64.4	43.4
0.6	160.8	155.6	147.5	141.0	136.1	126.9	108.9	76.3	44.1
0.7	129.6	126.6	123.4	117.9	112.4	108.6	102.0	84.4	49.2
0.8	99.2	97.6	95.9	93.9	91.6	87.6	84.1	80	59.4
0.9	79.6	74.9	71.4	69.2	68.0	66.9	65.6	63.2	60.4
1	78.9	72.35	66.8	62	57.9	54.3	51.1	48.3	45.7

Table 2.8: Minimal level of fixed entry cost, f^*, for which the dedicated incumbent is able to deter entry. Parameter values are θ=100, h =80, and $C_D = C_F$=10.

It seems counterintuitive that when the economic environment is very uncertain, an entrant that faces a flexible incumbent, prefers to invest in the dedicated capacity. However, in a highly uncertain economic environment it is very profitable for the incumbent to be the only firm in the market with flexibility. It will set its capacity so high that the entrant rather wants to invest in the dedicated production capacity, than later compete with the flexible incumbent in the production game. Therefore, for a fixed cost of entry slightly smaller than f^*, the incumbent faces a dedicated entrant[2].

[2] In case an intermediate levels of uncertainty is considered ($h = 40$), the incumbent can face a flexible or a dedicated incumbent, depending on the levels of substitutability and profitability of product 2. For low level of substitutability and low profitability of product 2, the incumbent faces in general a flexible entrant. For most parameter choices, entrant is indifferent between the

$\alpha \downarrow / \gamma \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	69.9	66.2	62.6	59.3	56.2	53.3	52.6	50	43.4
0.1	86.6	66.2	62.6	59.3	56.2	53.3	51.8	50.2	43.4
0.2	108.2	75.8	62.6	59.3	56.2	53.3	50.8	50.0	43.4
0.3	124.7	96.3	62.6	59.3	56.2	53.3	50.6	49.6	45
0.4	131	111.2	86.6	59.3	56.2	53.3	50.6	48.9	46.1
0.5	123.4	114	87	78.6	56.2	53.3	50.6	48.1	46.6
0.6	111.6	103.8	98.6	89.5	72	53.3	50.6	48.1	46.6
0.7	103	97.6	86.3	84.1	78.9	66.6	50.6	48.1	46.1
0.8	85.8	82.2	81.2	76.6	69.3	67.8	61.5	48.1	45.6
0.9	76.3	71.2	67.2	64.1	61.9	60.8	52.1	54.4	45.6
1	78.9	72.35	66.8	62	57.9	54.3	51.1	48.3	45.7

Table 2.9: Minimal level of fixed entry cost, f^*, for which the flexible incumbent is able to deter entry. Parameter values are θ=100, h =80, and $C_D = C_F$=10.

Comparing the results presented in Table 2.8 to Table 2.9 one can conclude that Result 2.3 does not hold in case of a highly uncertain economic environment. Given that demand uncertainty is high, it is more difficult for a dedicated incumbent to deter a flexible entrant, than for a flexible incumbent to deter a dedicated entrant.

Result 2.4

For a market with highly uncertain demand, a dedicated incumbent has to overinvest more than a flexible incumbent, to be able to deter the entrant, except when the profitability of product 2 is low and substitutability between the products is high.

Analyzing the differences of the results in Table 2.6 and Table 2.8 shows that in a market with high uncertainty it is harder for the dedicated incumbent to deter a flexible entrant. The high uncertainty makes the entrant a strong competitor, because the value of flexibility is high.

Comparison of Table 2.7 and 2.9, for the case where products are highly substitutable and profitability of product 2 is low, shows the same result. This

flexible and the dedicated production facility. For only a few parameter choices, the incumbent faces a dedicated entrant for a fixed entry cost slightly smaller than f^*. Here it is more difficult to deter a dedicated entrant that has a high revenue from the production of two products, where this strategy is not feasible for a flexible entrant (see Tables 2.10 and 2.11 in Appendix 2.C).

is the scenario where production of product 2 is the least appealing. The upper right corner of Table 2.7 shows that the minimum fixed cost of entry to deter is equal to f^*=43.4, in this scenario both firms produce product 1. A market environment that is very uncertain causes that product 2 is still an interesting production option for the incumbent. Therefore, in the upper right corner of Table 2.9, it is illustrated that the opportunity of the flexible incumbent to also produce product 2 makes it more difficult to deter the dedicated entrant that is fixed to its capacity of just product 1. This is another example of the value of commitment on the side of the entrant. However, the combination of a high profitability of product 2 and a low substitutability, is the best scenario for producing product 2, and therefore the flexible capacity is in favor for the incumbent. In this situation, a higher uncertainty leads to a lower minimum fixed cost of entry that is necessary to deter the dedicated entrant.

2.5 Conclusions

This chapter employs a three-stage game with two firms and two products considering uncertain demand. In the first stage, the incumbent invests in a product flexible or a dedicated production technology. The product flexible production technology gives the incumbent the opportunity to assign the available capacity freely to either one of the products. With the dedicated production technology, it will produce both products, each on a separate production line. In the second stage the entrant decides whether it will also invest in this market and if so, it chooses its optimal production technology and capacity level. After that, demand is revealed and a production game will be played in the last stage. The product flexible firm(s) optimize(s) the(ir) production amounts, given that the firms produce up to the capacity level.

Our results differ for two situations, a more uncertain and a less uncertain economic environment. When the economic environment is not very uncertain, we confirm the results of Anand and Girotra (2007), that investing dedicated can give an incumbent such a high ‘commitment value’, that the dedicated production technology is preferred over the product flexible production technology. Here the entrant always chooses for the product flexible technology. The commitment of the dedicated incumbent to its production quantities implies that the entrant has to optimize its production quantities given the (fixed) production quantities of the incumbent. Therefore, the incumbent needs to charge a lower fixed entry cost to deter the product flexible entrant with the dedicated production technology than with the product flexible production technology.

When the economic environment is more uncertain, the entrant chooses for the dedicated production technology. A higher demand uncertainty results in a higher value of flexibility that is large enough for the incumbent to prefer to be the only firm in the market profiting from the advantages of flexibility. Therefore, the incumbent makes a sufficiently large capacity investment that places the entrant in an unattractive production scenario. Therefore, the dedicated capacity is more profitable for the entrant, because it gives the entrant the first mover advantage in the production game.

Goyal and Netessine (2007) show that two dedicated firms can be the equilibrium if the firms simultaneously make their capacity and technology choice. However, we show that this situation will never occur when the firms make these choices sequentially. The only reason for the entrant to invest in the dedicated technology is that it gives the firm a first mover advantage in the production game. This advantage disappears when the incumbent already invested dedicated. When, however, the entrant faces a product flexible incumbent, we show that for some scenarios also the entrant can profit from the value of commitment. If the entrant uses the dedicated capacity, the incumbent cannot influence the entrant's production choice in the last stage. This results in a higher profit for the entrant compared to the choice of a product flexible capacity.

2.A Proof of Propositions

2.A.1 Proof of Proposition 2.1

For the case of strategy BD, considering Assumption 2.1, the corresponding expected profit of the entrant is equal to:

$$\mathbb{E}(\pi_{E,BD}) = 0.5\pi_{E,B} + 0.5\pi_{E,D} - C_F K_{F,E,BD} - f,$$

where

$$\pi_{E,B} = (\theta + h - (q_{1,E,B,BD} + K_{1,I}) - \gamma(q_{2,E,B,BD} + K_{2,I}))q_{1,E,B,BD} + \\ (\alpha(\theta + h) - (q_{2,E,B,BD} + K_{2,I}) - \gamma(q_{1,E,B,BD} + K_{1,I}))q_{2,E,B,BD}$$

$$\text{s.t. } q_{1,E,B,BD} + q_{2,E,B,BD} = K_{F,E,BD},$$

and

$$\pi_{E,D} = (\theta - h - (q_{1,E,D,BD} + K_{1,I}) - \gamma(q_{2,E,D,BD} + K_{2,I}))q_{1,E,D,BD} + \\ (\alpha(\theta - h) - (q_{2,E,D,BD} + K_{2,I}) - \gamma(q_{1,E,D,BD} + K_{1,I}))q_{2,E,D,BD}$$

$$\text{s.t. } q_{1,E,D,BD} + q_{2,E,D,BD} = K_{F,E,BD}.$$

Substituting the conditions $q_{1,E,B,BD} = K_{F,E,BD} - q_{2,E,B,BD}$ and $q_{1,E,D,BD} = K_{F,E,BD} - q_{2,E,D,BD}$ in the respective profit functions $\pi_{E,B}$ and $\pi_{E,D}$ gives the following expressions:

$$\pi_{E,B} = (\theta + h - (K_{F,E,BD} - q_{2,E,B,BD} + K_{1,I}) - \gamma(q_{2,E,B,BD} + K_{2,I}))(K_{F,E,BD} - q_{2,E,B,BD}) + \\ (\alpha(\theta + h) - (q_{2,E,B,BD} + K_{2,I}) - \gamma(K_{F,E,BD} - q_{2,E,B,BD} + K_{1,I}))q_{2,E,B,BD} \tag{2.A.1}$$

and

$$\pi_{E,D} = (\theta - h - (K_{F,E,BD} - q_{2,E,D,BD} + K_{1,I}) - \gamma(q_{2,E,D,BD} + K_{2,I}))(K_{F,E,BD} - q_{2,E,D,BD}) + \\ (\alpha(\theta - h) - (q_{2,E,D,BD} + K_{2,I}) - \gamma(K_{F,E,BD} - q_{2,E,D,BD} + K_{1,I}))q_{2,E,D,BD}. \tag{2.A.2}$$

We maximize both expression (2.A.1) and expression (2.A.2) with respect to the variables $q_{2,E,B,BD}$ and $q_{2,E,D,BD}$ respectively, and solve the obtained first order conditions simultaneously. This yields optimal production quantities $q^*_{2,E,B,BD}(K_{F,E,BD})$ and $q^*_{2,E,D,BD}(K_{F,E,BD})$, and we subsequently find $q^*_{1,E,B,BD}(K_{F,E,BD}) = K_{F,E,BD} - q^*_{2,E,B,BD}(K_{F,E,BD})$ and $q^*_{1,E,D,BD}(K_{F,E,BD}) = K_{F,E,BD} - q^*_{2,E,D,BD}(K_{F,E,BD})$. We maximize

$$\mathbb{E}(\pi_{E,BD}(q^*_{1,E,B,BD}(K_{F,E,BD}), q^*_{2,E,B,BD}(K_{F,E,BD}), q^*_{1,E,D,BD}(K_{F,E,BD}), q^*_{2,E,D,BD}(K_{F,E,BD})))$$

with respect to the capacity $K_{F,E,BD}$. Setting the first order condition equal to zero, and solving for $K_{F,E,BD}$ leads to the optimal capacity $K^*_{F,E,BD}$.

For the cases where the entrant uses strategy AD or AC, similar steps lead to the optimal quantities and capacities for the entrant. Remember that for strategy AD the entrant's quantity of product 2 is equal to zero, when the market goes 'up'. For strategy AC, the entrant's quantity of product 2 is equal to zero, for both 'up' and 'down' scenarios. □

Proof of Proposition 2.2 2.A.2

Consider the case where the entrant uses strategy BD. Under Assumption 2.1 and after substitution of the optimal quantities of the entrant, the expected profit function of the incumbent is equal to:

$$\mathbb{E}(\pi_{I,BD}) = 0.5\pi_{I,B} + 0.5\pi_{I,D} - C_D(K_{1,I} + K_{2,I}),$$

where,

$$\begin{aligned}\pi_{I,B} = {} & (\theta + h - (q^*_{1,E,B,BD} + K_{1,I}) - \gamma(q^*_{2,E,B,BD} + K_{2,I}))q^*_{1,E,B,BD} + \\ & (\alpha(\theta + h) - (q^*_{2,E,B,BD} + K_{2,I}) - \gamma(q^*_{1,E,B,BD} + K_{1,I}))q^*_{2,E,B,BD}\end{aligned}$$

and

$$\begin{aligned}\pi_{I,D} = {} & (\theta - h - (q^*_{1,E,D,BD} + K_{1,I}) - \gamma(q^*_{2,E,D,BD} + K_{2,I}))q^*_{1,E,D,BD} + \\ & (\alpha(\theta - h) - (q^*_{2,E,D,BD} + K_{2,I}) - \gamma(q^*_{1,E,D,BD} + K_{1,I}))q^*_{2,E,D,BD}.\end{aligned}$$

The optimal capacities can be derived by maximizing $\mathbb{E}(\pi_{I,BD})$ with respect to the capacities of the incumbent $(K_{1,I}, K_{2,I})$. The resulting optimal capacities are given by equations (3) and (4) in Proposition 2.2.

However, if those optimal capacities lie outside the area of strategy BD, the incumbent would choose a capacity level on the boundary of the region belonging to strategy BD. The boundary solution for the case that the production choice of product 2 of the entrant is equal to zero can be found by solving $q^*_{2,E,B,BD}(K_{1,I}, K_{2,I}, K_{2,E,BD}) = 0$ for $K_{2,E,BD}$. Then we substitute the optimal $K_{2,E,BD}$ into the entrant's expected profit function $\mathbb{E}(\pi_{I,BD})$. Next, we derive the first order condition of this expected profit function with respect to $K_{1,E}$, and solve for $K_{1,E,BD}$. As a result, $K^*_{1,E,BD}$ and $K^*_{2,E,BD}$ are the optimal capacities under the assumption that the production quantity of product 2 of the entrant is zero.

Similar steps lead to the optimal boundary solution for the case where either the capacity choice or the profit of the entrant is equal to zero. For those boundary solutions, we solve $K^*_{F,E,BD}(K_{1,I}, K_{2,I}) = 0$ and $\pi_{I,BD}(K_{1,I}, K_{2,I}) = 0$, respectively, for $K_{2,E,BD}$, and substitute into the expected profit function $\mathbb{E}(\pi_{I,BD})$. The next steps follow analogous to the previous case (that the production choice of the entrant is set equal to zero). $\square$

2.A.3 Proof of Proposition 2.3

For the case where the entrant faces scenario JN, considering Assumption 2.1, the corresponding expected profit of firm $j \in \{I, E\}$ is equal to:

$$\mathbb{E}(\pi_{j,JN}) = 0.5\pi_{j,J} + 0.5\pi_{j,N} - C_{F,j}K_{F,j,JN} - f,$$

where

$$\begin{aligned}\pi_{j,J} = (\theta + h - (q_{1,E,J,JN} + q_{1,I,J,JN}) - \gamma(q_{2,E,J,JN} + q_{2,I,J,JN}))q_{1,j,J,JN} + \\ (\alpha(\theta + h) - (q_{2,E,J,JN} + q_{2,I,J,JN}) - \gamma(q_{1,E,J,JN} + q_{1,I,J,JN}))q_{2,j,J,JN}\end{aligned}$$

$$\text{s.t. } q_{1,j,J,JN} + q_{2,j,J,JN} = K_{F,j,JN},$$

and

$$\pi_{j,N} = (\theta + h - (q_{1,E,N,JN} + q_{1,I,N,JN}) - \gamma(q_{2,E,N,JN} + q_{2,I,N,JN}))q_{1,j,N,JN} +$$

$$(\alpha(\theta+h)-(q_{2,E,N,JN}+q_{2,I,N,JN})-\gamma(q_{1,E,N,JN}+q_{1,I,N,JN}))q_{2,j,N,JN}$$

$$\text{s.t. } q_{1,j,N,JN}+q_{2,j,N,JN}=K_{F,j,JN}.$$

Substituting the conditions $q_{1,j,J,JN}=K_{F,j,JN}-q_{2,j,J,JN}$ and $q_{1,j,N,JN}=K_{F,j,JN}-q_{2,j,N,JN}$ in the respective profit functions $\pi_{E,J}$ and $\pi_{E,N}$ gives the following expressions:

$$\begin{aligned}\pi_{j,J}=&\Big(\theta+h-(K_{F,E,JN}-q_{2,E,J,JN}+K_{F,I,JN}-q_{2,I,J,JN})\\&\quad-\gamma(q_{2,E,J,JN}+q_{2,I,J,JN})\Big)(K_{F,j,JN}-q_{2,j,J,JN})\\&+\Big(\alpha(\theta+h)-(q_{2,E,J,JN}+q_{2,I,J,JN})\\&\quad-\gamma(K_{F,E,JN}-q_{2,E,J,JN}+K_{F,I,JN}-q_{2,I,J,JN})\Big)q_{2,j,J,JN},\end{aligned} \tag{2.A.3}$$

and

$$\begin{aligned}\pi_{j,N}=&\Big(\theta+h-(K_{F,E,JN}-q_{2,E,N,JN}+K_{F,I,JN}-q_{2,I,N,JN})\\&\quad-\gamma(q_{2,E,N,JN}+q_{2,I,N,JN})\Big)(K_{F,j,JN}-q_{2,j,N,JN})\\&+\Big(\alpha(\theta+h)-(q_{2,E,N,JN}+q_{2,I,N,JN})\\&\quad-\gamma(K_{F,E,JN}-q_{2,E,N,JN}+K_{F,I,JN}-q_{2,I,N,JN})\Big)q_{2,j,N,JN}.\end{aligned} \tag{2.A.4}$$

We maximize both expression (2.A.3) and expression (2.A.4) with respect to the variables $q_{2,j,J,JN}$ and $q_{2,j,N,JN}$ respectively, for $j\in\{I,E\}$. The obtained first order conditions have to be solved simultaneously for both firms, which yields optimal production quantities $q^*_{2,j,J,JN}(K_{F,j,JN})$ and $q^*_{2,j,N,JN}(K_{F,j,JN})$, and we subsequently find $q^*_{1,j,J,JN}(K_{F,j,JN})=K_{F,j,JN}-q^*_{2,j,J,JN}(K_{F,j,JN})$ and $q^*_{1,j,N,JN}(K_{F,j,JN})=K_{F,j,JN}-q^*_{2,j,N,JN}(K_{F,j,JN})$. We maximize, for both firms $j\in\{I,E\}$,

$$\mathbb{E}(\pi_{j,JN}(q^*_{1,j,J,JN}(K_{F,j,JN}),q^*_{2,j,J,JN}(K_{F,j,JN}),q^*_{1,j,N,JN}(K_{F,j,JN}),q^*_{2,j,N,JN}(K_{F,j,JN})))$$

with respect to the capacity $K_{F,j,JN}$. Setting the first order condition equal to zero, and solving simultaneously for $K_{F,E,JN}$ and $K_{F,I,JN}$ leads to optimal capacities $K^*_{F,E,JN}$ and $K^*_{F,I,JN}$.

Expressions of the optimal quantities and capacities for the other cases are obtained similarly. □

2.A.4 Proof of Proposition 2.4

We will prove in two steps, that it is a dominant strategy for the entrant to invest in the flexible capacity, when it observes an incumbent with a dedicated capacity. First we show that a dedicated investment of the entrant (strategy EF) in two products is dominated by the strategy BD (see Figure 2) when it would invest in the flexible capacity. Second, we show that a dedicated investment in only one product is dominated by strategy AC of a flexible entrant.

First, assume that both firms invest in the dedicated production technology (strategy EF), and that it is optimal for the entrant to invest in a positive amount of both products. In this case, the profit of the entrant is equal to

$$\pi_{E,EF} = \frac{(C_F^2 - (1+\alpha)C_F\theta)(1-\gamma) + 0.5(1+\alpha^2 - 2\alpha\gamma)\theta^2}{8(1-\gamma^2)}.$$

When only the incumbent invests in the dedicated capacity and the entrant invests in the flexible capacity, producing both products in both scenarios (strategy BD) gives the entrant a profit equal to

$$\pi_{E,BD} = \frac{(C_F^2 - (1+\alpha)C_F\theta)(1-\gamma) + (1-\alpha)^2h^2(1+\gamma) + 0.5(1+\alpha^2 - 2\alpha\gamma)\theta^2}{8(1-\gamma^2)}.$$

Comparison of these profits show that $\pi_{E,EF} < \pi_{E,BD}$. Therefore, when both situations are feasible, the entrant will always obtain a higher profit from investment in the flexible capacity. Next, we show that when there exists a nonempty region for strategy BD in the $(K_{1,I}, K_{2,I})$-plane, the feasible region for strategy EF will be a subset of the region belonging to strategy BD. A strategy is feasible when it implies positive capacities, and positive production quantities. We find that strategy EF and BD result in the same total optimal capacity:

$$K_{1,E,EF} + K_{2,E,EF} = K_{F,E,BD} = \frac{(1+\alpha)\theta - (1+\gamma)(K_{1,I} + K_{2,I}) - 2C_F}{2(1+\gamma)}.$$

Solving $q_{2,E,B,BD}(K_{1,I}, K_{2,I}) = 0$ for $K_{2,I}$ results in boundary $K_{2,I}^{BD}$ for the region belonging to strategy BD. Similarly, solving $K_{2,E,EF}(K_{1,I}, K_{2,I}) = 0$ for $K_{2,I}$ results in boundary $K_{2,I}^{EF}$ for the region belonging to strategy EF. Then, capacity

$K_{2,E,EF}(K_{1,I}, K_{2,I})$ and quantity $q_{2,E,B,BD}(K_{1,I}, K_{2,I})$ are given by

$$K_{2,E,EF}(K_{1,I}, K_{2,I}) = \frac{\theta(\alpha-\gamma)-(1-\gamma^2)K_{2,I}-(1-\gamma)C_F}{2(1-\gamma^2)},$$

$$q_{2,E,B,BD}(K_{1,I}, K_{2,I}) = \frac{\theta(\alpha-\gamma)-(1-\gamma^2)K_{2,I}-(1-\gamma)C_F-0.5(1-\alpha)(1+\gamma)h}{2(1-\gamma^2)}.$$

Because $K_{2,E,EF}(K_{1,I}, K_{2,I}) > q_{2,E,B,BD}(K_{1,I}, K_{2,I})$, it holds that $K_{2,I}^{BD} > K_{2,I}^{EF}$. Therefore the feasible region for strategy EF is a subset of the feasible region for strategy BD. Together with the fact that strategy BD always gives a higher profit than strategy EF for the entrant, we can conclude that strategy BD dominates strategy EF.

Second, assume that it is optimal for a dedicated entrant to invest only in product 1. When both firms invest in the dedicated production technology, the profit of the entrant is equal to

$$\pi_{E,EF} = \frac{1}{16}(\theta - C_D)^2.$$

When only the entrant invests in the flexible production technology, and produces only product 1 in the up- and downward scenario (strategy AC), the profit of the entrant is equal to

$$\pi_{E,AC} = \frac{1}{16}(\theta - 3C_D + 2C_F)^2.$$

Since we consider the case where investment costs per unit of capacity are equal for the two types of capacities, we find that $\pi_{E,EF} = \pi_{E,AC}$. Therefore, for the case where it is optimal for the entrant to invest only in the dedicated capacity of product 1, it is indifferent between investing dedicated and investing flexible (with strategy AC). For the parameter combinations for which it is only feasible for the entrant to invest in only the dedicated capacity of product 1, it might be feasible with the flexible capacity to invest in strategy BD, AD or AC. We have shown that the entrant's profit with strategy AC is equal to strategy EF. Performing strategy BD or AD implies that the entrant will also optimally produce a positive amount of product 2, therefore the entrant's profit corresponding to these strategies are bigger than from strategy AC. This indicates that an entrant will always obtain a higher profit from investing in the flexible production technology, when it observes a dedicated incumbent. □

2.B Additional Proofs

2.B.1 Stochastic discount factor equal to h

Lemma 2.1
The standard deviation of the price intercept at time $t = 2$ is equal to h.

Proof
Define the price intercept at time $t = k$ by $\theta_{t=k}$ for $k \in \{0, 1, 2\}$. At time $t = 0$ and $t = 1$ demand is still uncertain, only at time $t = 2$ demand uncertainty is resolved. We find that

$$\theta_{t=2} = \begin{cases} \theta_{t=0} + h & \text{with} \quad p = \frac{1}{2}, \\ \theta_{t=0} - h & \text{with} \quad 1 - p = \frac{1}{2}. \end{cases}$$

Therefore,

$$\mathbb{E}[\theta_{t=2}] = \theta_{t=0},$$

and

$$\mathbb{E}[\theta_{t=2}^2] = \theta_{t=0}^2 + h^2,$$

such that

$$\text{Var}(\theta_{t=2}) = h^2.$$

□

2.B.2 Elimination of Strategy BC

Lemma 2.2
Production strategy BC of the entrant (i.e. produce two products when the market

makes an upward move, and one product when the market moves downward) is an infeasible strategy.

Proof

We first show that the following inequality holds:

$$\mathbb{E}(\pi_{E,BD}(K_{1,I},K_{2,I}))-\mathbb{E}(\pi_{E,BC}(K_{1,I},K_{2,I}))\geq 0 \qquad \forall(K_{1,I},K_{2,I}). \tag{2.B.1}$$

We take the first order derivatives of $\mathbb{E}(\pi_{E,BD}(K_{1,I},K_{2,I}))-\mathbb{E}(\pi_{E,BC}(K_{1,I},K_{2,I}))$ with respect to $K_{1,I}$ and $K_{2,I}$. The first order derivative with respect to $K_{1,I}$ is zero. We set the derivative with respect to $K_{2,I}$ equal to zero, in order to derive the stationary points. Then we show that the difference in these expected profit function is convex. The stationary points are given by:

$$(K_{1,I}^*,K_{2,I}^*)=\left(K_{1,I},\frac{\theta(\alpha-\gamma)+\frac{1}{2}(1-\alpha)(1+\gamma)h-C_F(1-\gamma)}{1-\gamma^2}\right), \tag{2.B.2}$$

s.t. $K_{1,I}\geq 0$.

The Hessian matrix of the function $\mathbb{E}(\pi_{E,BD}(K_{1,I},K_{2,I}))-\mathbb{E}(\pi_{E,BC}(K_{1,I},K_{2,I}))$ is given by:

$$H\left(\mathbb{E}(\pi_{E,BD}(K_{1,I},K_{2,I}))-\mathbb{E}(\pi_{E,BC}(K_{1,I},K_{2,I}))\right)=\begin{bmatrix} 0 & 0 \\ 0 & \frac{1-\gamma^2}{3+\gamma} \end{bmatrix} \tag{2.B.3}$$

One can easily see that it is positive-semidefinit. Consequently, this function is convex and the minimum location is given by the expressions in equation (2.B.2). Substituting those into $\mathbb{E}(\pi_{E,BD})-\mathbb{E}(\pi_{E,BC})$ gives:

$$\mathbb{E}(\pi_{E,BD}(K_{1,I},K_{2,I}^*))-\mathbb{E}(\pi_{E,BC}(K_{1,I},K_{2,I}^*))=0.$$

Since $\mathbb{E}(\pi_{E,BD})-\mathbb{E}(\pi_{E,BC})$ is convex and the inequality of expression (2.B.1) holds in the minimum location, we conclude that the inequality of expression (2.B.1) holds for all capacity choices of the incumbent, i.e. the expected profit from strategy BD is always higher than the expected profit the entrant would get choosing for strategy BC.

However, strategy BD is not always feasible. When the entrant plays strategy BD, it will always produce both products, no matter if the market goes 'up' or 'down'. The optimal quantities of product 2 should not be negative, otherwise strategy BD would be infeasible. If the market goes 'up', the production quantity of product 2 will always be less than the production quantity of product 2 when the market goes 'down', i.e. $q_{2,E,B,BD} < q_{2,E,D,BD}$, where $q_{2,E,B,BD}$ is the production quantity of product 2 of the entrant that plays strategy BD (and situation B arises), when the market goes 'up'. Consequently, regarding the optimal quantities of strategy BD it only needs to be checked whether $q_{2,E,B,BD} > 0$ for this strategy to be feasible.

Strategy BC can only be the best feasible strategy if there exists a situation where strategy BD is not feasible and strategy BC is feasible (and better than all other strategies). We will show below that for all situations for which strategy BC is feasible, strategy BD will also be feasible. Consequently, strategy BC will never be chosen because strategy BD will always give a higher profit.

First we derive the thresholds for which strategies BD and BC are on the boundary of feasibility/ infeasibility. For convenience, the boundaries have been given names.

For $K_{2,I} \leq \frac{\theta(\alpha-\gamma)}{1-\gamma^2} - \frac{h(1-\alpha)}{(1-\gamma^2)} - \frac{C_F}{1+\gamma} = \widehat{q_{2,E,B,BC}}$, strategy BC has a positive quantity of product 2 in case situation B occurs. This is a necessary condition for strategy BC to be feasible.

For $K_{2,I} \leq \frac{\theta(\alpha-\gamma)}{1-\gamma^2} - \frac{h(1-\alpha)\frac{1}{2}}{(1-\gamma)} - \frac{C_F}{1+\gamma} = \widehat{q_{2,E,B,BD}}$, strategy BD has a positive quantity of product 2 in case situation B occurs. This is a necessary condition for strategy BD to be feasible.

Next we show that $\widehat{q_{2,E,B,BC}} < \widehat{q_{2,E,B,BD}}$:

$$\widehat{q_{2,E,B,BC}} < \widehat{q_{2,E,B,BD}} \Leftrightarrow -\frac{h(1-\alpha)}{(1-\gamma^2)} < -\frac{h(1-\alpha)\frac{1}{2}}{(1-\gamma)} \Leftrightarrow \frac{h(1-\alpha)}{(1-\gamma^2)} > \frac{h(1-\alpha)\frac{1}{2}}{(1-\gamma)}$$
$$\Leftrightarrow \frac{1}{(1+\gamma)} > \frac{1}{2} \Leftrightarrow 2 > 1+\gamma \Leftrightarrow 1 > \gamma.$$

Therefore, it holds that the range of values of $K_{2,I}$ for which strategy BC is feasible is smaller than for strategy BD. □

2.C Additional Tables

Tables 2.10 and 2.11 illustrate the minimal level f of fixed entry cost that leads to entry deterrence in case of a dedicated and flexible incumbent, respectively,

$\alpha\downarrow/\gamma\rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	79.7	57.5	44.4	43.4	43.4	43.4	43.4	43.4	43.4
0.1	91.8	72.6	52.2	43.4	43.4	43.4	43.4	43.4	43.4
0.2	98.0	83.9	66.2	47.8	43.4	43.4	43.4	43.4	43.4
0.3	98.0	89.1	76.8	60.3	44.8	43.4	43.4	43.4	43.4
0.4	97.2	88.9	81.3	70.5	54.9	43.4	43.4	43.4	43.4
0.5	93.0	87.9	80.6	74.1	64.8	49.7	43.4	43.4	43.4
0.6	86.0	82.8	78.9	73.4	67.5	69.6	45.4	43.4	43.4
0.7	77.5	75.5	73.2	70.5	66.8	61.5	55.0	43.4	43.42
0.8	72.5	68.5	65.7	64.0	62.2	60.0	56.1	50.8	43.4
0.9	72.9	67.5	62.9	59.1	56.2	54.2	52.8	51.0	46.9
1	78.9	72.35	66.8	62	57.9	54.3	51.1	43.4	43.4

Table 2.10: Minimal level of fixed entry cost, f^*, for which the dedicated incumbent is able to deter entry. Parameter values are θ=100, h =40, and $C_D = C_F$=10.

$\alpha\downarrow/\gamma\rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	104.5	74.2	70.9	65.8	58.2	47.0	43.4	43.4	43.4
0.1	118.6	91.2	69.9	66.3	60.5	51.5	43.4	43.4	43.4
0.2	127.3	104.6	67.8	65.8	61.8	54.9	43.4	43.4	43.4
0.3	133.3	112.5	92.5	64.2	61.8	57.0	48.3	43.4	43.4
0.4	131.7	117.4	99.9	82.1	60.6	57.8	51.8	43.4	43.4
0.5	120.5	113.9	103.4	88.9	73.2	57.1	53.6	43.4	43.4
0.6	110.0	102.1	97.9	91.1	79.3	65.5	53.7	48.9	43.4
0.7	102.2	94.4	88.1	82.3	78.9	70.8	58.8	50.2	43.4
0.8	87.9	83.1	79.4	75.6	70.8	67.1	62.8	53.0	46.1
0.9	76.7	72.2	68.6	65.3	62.3	59.9	56.9	54.2	47.9
1	78.9	72.35	66.8	62	57.9	54.3	51.1	43.4	43.4

Table 2.11: Minimal level of fixed entry cost, f^*, for which the flexible incumbent is able to deter entry. Parameter values are θ=100, h =40, and $C_D = C_F$=10.

for an intermediate level of uncertainty (h=40). If the incumbent invests flexible, it can face a dedicated and a flexible entrant, depending on the level of substitutability and profitability of product 2 (see Table 2.11). The parameter combinations that result in a scenario with a flexible entrant are shaded in darker grey and the parameter combinations that result in a dedicated entrant in lighter grey. The entries in Table 2.11 are unshaded for the parameters where the entrant is indifferent between the flexible or the dedicated capacity.

3 Sensitivity of Demand Function Choice in a Strategic Real Options Context

This chapter is based on Boonman and Hagspiel (2014).

Abstract

This chapter studies the effect of three commonly used demand functions on the investment decisions of two potential entrants in a new market model. Two of these demand functions are linear and known in the literature as additive and multiplicative demand. The third is the iso-elastic demand function.

We show that the result of the second firm investing in a larger capacity size than the first investor is mainly robust concerning the choice of demand model. However, in case the economic environment is very uncertain, there are high fixed costs with the investment and if the demand is multiplicative, the capacity of the first investor is larger. Furthermore, when variable investment costs increase, for the additive demand case we find that firms invest later and in a larger capacity, whereas for the multiplicative and iso-elastic demand functions the firms also invest later but then in less capacity. We also find under exogenous firm roles that for the additive demand function the first investor always applies an entry deterrence strategy.

3.1 Introduction

In the early nineties, the fast food chain Boston Chicken was a good business. The firm added restaurants at a staggering rate, resulting in an average growth rate of 82% per year. However, by 1998, the firm grew so fast that it was unable to maintain the quality expectations of consumers, and could not compete against the increasing numbers of rival firms. Later that year, the fast food chain had to file for bankruptcy in 1998 (Ross et al. (2006)). This example highlights the necessity of firms carefully estimating the size and growth of the potential market, and thereupon adapting what they believe about their demand structure. The outcome for Boston Chicken might have been different if it had realized that the potential market was limited and had to be shared with its competitors.

This chapter wants to emphasize the importance of choosing the correct demand structure when evaluating investment decisions, under the general framework of competition, capacity optimization and optimal timing of investment. We consider a duopoly game where the firms produce a single homogeneous type of goods. The optimal level of capacity is chosen alongside the optimal timing decision. The market leader has the advantage to use its capacity size to deter the entrant, and maintain its monopoly situation a little longer.

When capacity or quantity optimization becomes an issue, one has to specify the demand function in more detail. Most papers in this field only introduce a specific type of demand function, but refrain from explaining their choice. Aguerrevere (2003) and Kulatilaka and Perotti (1998) study optimal capacity investment in a real options setting under the assumption of a demand function with a linear *additive demand shock*. The additive demand structure is defined by $P_t = A_t - \eta Q_t$, where P_t denotes the price at time t, Q_t the demand, and A_t the demand shock parameter that follows a specified distribution. Grenadier (2000) and Goto et al. (2008) are examples that consider competition in a real options setting using a *multiplicative demand shock* structure. They define their price function as the demand shock multiplied by a function that depends on demand: $P_t = A_t * D(Q_t)$. Similar to Huisman and Kort (2014), we define $D(Q_t)$ more specifically by assuming that $D(Q_t) = 1 - \eta Q_t$. Furthermore, one can categorize a third major type of demand function, which is a specific form of the multiplicative demand function, i.e. the *iso-elastic demand shock*. This model is used, in Aguerrevere (2009) and Marx (2007). The iso-elastic demand model is defined by $P_t = A_t Q_t^{-1/\gamma}$, with $\gamma > 1$ being the elasticity of demand.

To our knowledge, there are two papers that explicitly address the impact of the demand function choice on optimal investment decisions. Anupindi and Jiang (2008) categorizes demand functions into two streams for different demand shocks, multiplicative and additive. However, their results with respect to the two types of demand shocks focus on the comparison of a dedicated and flexible production technology and not on determining the optimal capacity size. Despite considering stochastic demand, their model does not allow them to derive the optimal timing of investment either. Furthermore, their approach differs from ours by considering symmetric firms, while this paper takes into account a leader-follower setting. Ming-Gao et al. (2011) extends the work of Fontes (2008) that distinguishes between three types of flexibility (i.e. contract capacity - switch to a lower capacity level; expand capacity - switch to an upper capacity level; or both contract and expand capacity), by comparing results for the multiplicative and additive demand function. They find that the capacity flexibility premium is significantly higher under the additive demand function.

While those papers emphasize that the choice of a specific demand function can have a significant impact on their results, we have not found a paper that elaborates on the explicit impact of the demand function choice on the timing and capacity of an optimal investment decision. This is where we want to make a contribution to the literature. We study the effect of three commonly used demand functions, i.e. additive, multiplicative and iso-elastic, on the optimal investment decisions of two competing firms. Initially, we study the firm's investment decision under exogenous (i.e. predetermined) firm roles. Here, the leader can use two strategies, entry deterrence and entry accommodation. Entry deterrence can be achieved by overinvestment. This gives the leader a period of monopoly profits. Under entry accommodation, overinvestment is too expensive and it allows the entrant to invest simultaneously with the leader. Later we study endogenous firm roles, where the firms compete for the position of leader. By construction, this results in an entry deterrence situation.

Where we assume a duopoly model, other market structures such as a monopoly or oligopoly are also discussed in the literature. The monopolistic model of Manne (1961) lies at the heart of the capacity management literature, which initially focussed on capacity expansion. It finds that installed capacity should be of a larger size under uncertainty compared to the deterministic case. Hagspiel (2011) and Dangl (1999) confirm this result, showing that increasing uncertainty does not just lead to investment in larger capacity but also delays investment. Yang and Zhou (2007) based their research on the framework of Dangl (1999) and extend this additive demand model by introducing duopoly firms. Similar to what Huisman and Kort (2014) found for a multiplicative demand structure, their results show that the entrant cannot be deterred by investment in excess capacity. Eventually, the entrant will invest in the market. Where this literature analyzes monopoly and duopoly models, Bouis et al. (2009) emphasizes the importance of a multiple decision-maker framework (i.e. an oligopoly). Papers that take up this challenge are Kulatilaka and Perotti (1998) and Grenadier (2000). The latter explains that the impact of competition drastically erodes the value of the option to wait and leads to investment at a close to zero net present value threshold. For a further review on strategic investment under uncertainty, we refer to Chevalier-Roigant et al. (2011).

In the industrial organization literature (Tirole (1988)), entry deterrence and entry accommodation are concepts that are typically known in two-period models, where in the first period the incumbent makes its investment decision and in the second period the potential entrant reacts. By overinvesting, the incumbent can achieve entry deterrence, which results in a market where the entrant refrains from entering. Similar to Huisman and Kort (2014), we extend this concept to a continuous time framework. In this model, entry deterrence implies that the entrant invests later than the incumbent, which results in a period of monopoly profit for the leader. With entry accommodation we refer to the situation where both firms invest at the same time.

We show that the choice of demand function can have a significant impact on the results. Which explains the fact that this decision has to be made carefully. We find that under multiplicative demand, the leader invests in a larger capacity than the follower when the economic environment is very uncertain, and the fixed costs of investment are sufficiently low. Under additive and iso-elastic demand however, the opposite result occurs, i.e. the follower always invests in a larger capacity than the leader.

In order to understand these results, we need to understand the implications that the structure of the demand function has on the corresponding market size. The multiplicative demand function has a fixed price intercept, which implies that there is an upper bound on the market size. Consider for instance the sale of agricultural machines, e.g. harvesters, in a region like the Netherlands where the amount of acres and farmers is limited. This results in an upper bound on demand. Another example of a market where there is a limited potential market size is the market of medical equipment. The amount of medical staff performing their profession is restricted, which also restricts the sale of medical equipment. However, this holds for many professions that are depending on their working tools. The additive and iso-elastic demand function on the other hand, do not pose an upper bound on the market size. This relates to most of the product markets. A good example is the online shopping branch, which has been flourishing for quite some years, and is still expected to continue its growth.

Using this knowledge, we can now understand our results. The multiplicative demand function poses an upper bound on the market size, which has to be shared among the two firms in the market. The leader can make its investment decision first, which in combination with the fixed market size leads to the result that it can achieve the largest capacity in the market. The negative relationship between market entry and market share is supported by the majority of related empirical literature (e.g. Kalyanaram et al. (1995), Uran et al. (1986) and Kalyanaram and Wittink (1994)). The opposite result occurs for the additive and iso-elastic demand. The advantage of waiting for the follower's position is that it can observe the demand realization and thereby receive more information about the market. Delaying the investment decision corresponds to investment in a larger capacity (e.g. Dangl (1999), Bar-Ilan and Strange (1999)). Without the cap on the market size, the follower faces a market of larger size at the moment of investment, which explains its larger capacity choice. An example that corresponds to the results obtained with the iso-elastic and additive demand is the e-commerce company Zalando GmbH (founded in 2008 in Germany). It began operations abroad by starting offering deliveries in Austria in 2009 and in the Netherlands and France in 2010. In France, the competitors at that moment included Spartoo, Sarenza, Otto and Asos[1]. Due to an extensive and effective marketing campaign the new entrant Zalando became the market leader in online shoe sales in France, only one year after entry. Uran et al. (1986) gives another example in the freeze-dried coffee market, Maxwell House's Maxim pioneered the category, but Nestle's Taster's Choice identified a superior position and overtook Maxim. Notice that both

[1]http://www.eu-startups.com/2011/07/european-shoe-business-zalando-vs-spartoo/

examples describe a market that has no obvious restriction in its size. Lieberman and Montgomery (1988) point out that, besides this effect, there are several benefits which place late movers in a favorable position, i.e. "(1) the ability to free-ride on first mover's investments, (2) resolution of technological and market uncertainty, (3) technological discontinuities that provide 'gateways' for new entry, and (4) various types of 'incumbent inertia' that makes it difficult for the incumbent to adapt to environmental change". Notice however, that our model only supports the argument about the resolution of market uncertainty.

Additionally we show that the use of the additive demand structure leads to a different effect of an increase in the variable investment cost compared to the multiplicative and iso-elastic demand structure. The results can be explained by use of a direct and an indirect effect. A slightly higher variable cost decreases the optimal capacity (direct effect), but also delays investment which corresponds to a larger capacity amount (indirect effect). Only under the additive demand structure, a firm faces a higher market size when waiting with investment to obtain more information about the market. This is thereby also an extra incentive for a firm to delay its investment decision and justify the large capacity choice, i.e. the indirect effect dominates. Even though the iso-elastic demand structure has no fixed market size, like the multiplicative demand structure, it is not unbounded. A high capacity choice results in a very low price of the product, which makes it undesirable for a firm to choose such level of capacity. Therefore, for the multiplicative and iso-elastic demand function the direct effect dominates due to a limitation in the market size.

The rest of this chapter is structured as follows. The general model is presented in the Section 3.2. Section 3.3 analyzes the capacity and timing decision of the two firms, where Section 3.3.1 considers the additive linear demand function, and Section 3.3.2 treats the multiplicative linear demand function. In Section 3.4 the iso-elastic demand function is compared to the two linear demand functions. Section 3.5 concludes.

3.2 Model

We consider a duopoly game, where two firms produce a single homogeneous type of goods. The firms are assumed to be risk neutral and value maximizing, and future profits are discounted with constant discount rate r. Each firm is able to undertake an irreversible investment at one point in time. We consider three different types of inverse demand functions[2]. These inverse demand functions are defined by:

$$P_{t,mult} = X_t(1 - \eta Q_t), \tag{3.2.1}$$

for multiplicative demand,

$$P_{t,add} = X_t - \eta Q_t, \tag{3.2.2}$$

for additive demand, and

$$P_{t,iso} = X_t Q_t^{-\gamma}, \tag{3.2.3}$$

for iso-elastic demand at time t, where the total market output is denoted by Q_t. $\eta > 0$ and $1 > \gamma > 0$ are constants. Demand uncertainty is modeled by the process $\{X_t\}$ following the geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t d\omega_t,$$

with the drift parameter μ, σ the volatility parameter, and $d\omega_t$ the increment of a standard Wiener process. We assume that $r > \mu$, otherwise it would never be optimal to invest. The investment costs are modeled similar to Aguerrevere (2003). A firm that enters the market with capacity size Q_t faces investment costs equal to $\delta_1 Q_t + \delta_0$, where δ_1 denotes the sensitivity of investment costs with respect to capacity size and δ_0 the fixed investment costs. For analytical convenience, it is assumed in Section 3.3 that $\delta_0 = 0$, leaving us with the linear cost model. Here we compare the investment decision, in a duopoly setting, for a model with multiplicative and additive demand. In Section 3.4 we consider the general case, with $\delta_0 > 0$. Including constant investment costs enables us to numerically compare the linear demand models with the iso-elastic demand model. Since the use of linear investment costs with $\delta_0 = 0$ in an iso-elastic

[2]Under optimal capacity, these demand structures have a different price development. Where the iso-elastic demand structure results in a flat price, under the multiplicative and additive demand structure, the price is increasing.

demand model results in a profit function that is linear in capacity Q_t, one cannot find a finite solution for optimal capacity size Q_t.

We impose a necessary assumption with its justification:

Assumption 3.1
Both firms produce up to capacity.

Assumption 3.1, often called the 'market clearance assumption', is widely used in the literature (Chod and Rudi (2005), Deneckere et al. (1997), Anand and Girotra (2007), and Goyal and Netessine (2007)). Fixed costs, like commitments to suppliers and production ramp-up might make it too costly to produce below the capacity level (Goyal and Netessine (2007)). An example is the car industry where firms often decide to rather cut prices and keep production up to full capacity, than produce below capacity (Mackintosh (2003)). Additional reasons are strict labor laws that prevent employees from easily getting fired. Especially in countries with strict laws, companies often have to pay a considerable amount of money for letting employees go. Besides that, knowledge will be lost. The American Federal Reserve provides detailed information about US capacity utilization rates on their website[3] and shows that these fluctuate around the eighty percent. The market clearance assumption is posed for analytical convenience, however this evidence reveals that it lies relatively close to reality.

3.3 Linear Investment Costs

In this section we consider a duopoly, consisting of a leader and a follower[4]. We denote the optimal capacity of the leading and following firm by Q_L and Q_F, respectively. Once both investors have invested, the total market output is equal to $Q = Q_L + Q_F$. In this section we consider investment costs being equal to $\delta_1 Q$. At first, the roles of the firms are assumed to be exogenous. In other words, the leading firm knows that it will be the first investor in the market and cannot be preempted by the other firm. The other firm is the follower, which

[3] www.federalreserve.gov

[4] For the monopoly case, the monopolist's investment decision is equal to the follower's investment decision, where the optimal capacity of the leader is equal to zero. Huisman and Kort (2014) analyzes the monopoly scenario for the multiplicative demand function.

knows that it will have to wait with its investment until the leader has invested. The obtained payoffs in the exogenous firm roles case form the basis of our analysis of the model with endogenous firm roles, where the market position of the two firms is not predefined. This section is divided into two subsections, where we will discuss the effect of the two different types of linear demand functions.

Additive Demand 3.3.1

In this section we consider the additive inverse demand function, defined by (3.2.2). We assume that the initial level of demand shock is low enough to fall below any of the investment triggers derived in this section. We solve this game backwards, which implies that we start by deriving the optimal investment decision of the follower, later we consider the investment decisions of the leader. The follower invests when the leader is already in the market, and therefore the optimal decision of the leading firm is known to the follower. Consequently, the optimal investment timing ($X_{F,add}(Q_L)$) and the optimal capacity ($Q_{F,add}(Q_L)$) of the follower are functions of the optimal leader's capacity Q_L. The optimal capacity and investment thresholds of the follower are the content of Proposition 3.1.

Proposition 3.1
Suppose the additive inverse demand function $P_{t,add} = X_t - \eta Q_t$. Given the current level of stochastic demand shock X and the leader's capacity level Q_L, the optimal capacity level of the follower $Q_{F,add}(X, Q_L)$ is equal to:

$$Q_{F,add}(X, Q_L) = \frac{Xr - (r-\mu)(r\delta_1 + Q_L\eta)}{2\eta(r-\mu)}. \tag{3.3.1}$$

The value function of the follower $V_F(X, Q_L)$ is given by

$$V_{F,add}(X, Q_L) = \begin{cases} A_{F,add}(Q_L)X^{\beta} & \text{if } X < X_{F,add}(Q_L), \\ \frac{(Xr-(r-\mu)(r\delta_1+Q_L\eta))^2}{4r\eta(r-\mu)^2} & \text{if } X > X_{F,add}(Q_L), \end{cases}$$

where

$$A_{F,add}(Q_L) = \left(\frac{r(\beta-2)}{\beta(r\delta_1 + Q_L\eta)(r-\mu)}\right)^{\beta}\left(\frac{(r\delta_1 + Q_L\eta)^2}{(\beta-2)^2 r\eta}\right),$$

$$X_{F,add}(Q_L) = \frac{\beta(r\delta_1 + Q_L\eta)(r-\mu)}{r(\beta-2)}, \tag{3.3.2}$$

and β the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0,$$

so that

$$Q_{F,add}(Q_L) = Q_{F,add}(X_{F,add}(Q_L), Q_L) = \frac{r\delta_1 + Q_L\eta}{r(\beta-2)}.$$

For the same reason as before, we assume that $r > 2\mu + \sigma^2$. That is, $\beta > 2$. Otherwise it would never be optimal to invest.

In the next step we determine the investment decision of the leader. The leader has two possible strategies. Entry deterrence corresponds to sequential investments and gives the leading firm a monopoly profit for a period of time starting right after its investment, till the moment where the follower enters. Entry accommodation leads to simultaneous investments, where the follower invests at the same time as the leader. The leader uses its optimal capacity Q_L as a tool to force either one of those strategies. It follows from equation (3.3.2) that a deterrence strategy occurs when the leader invests in a capacity size that is larger than Q_t. It follows from equation (3.3.2) that the deterrence strategy occurs when the leader invests in a capacity size Q_L that is larger than $\widehat{Q_{L,add}}(X)$, such that

$$\widehat{Q_{L,add}}(X) = \frac{r\left(X(\beta-2) - \delta_1(r-\mu)\beta\right)}{(r-\mu)\beta\eta}, \tag{3.3.3}$$

Notice that $X_{F,add}(Q_L)$ is increasing in Q_L, i.e. the leader can extend its monopoly period by investing in a larger capacity. Besides delaying investment of the follower, another incentive for the leader to invest in a large capacity is that the capacity $Q_{F,add}(X, Q_L)$ decreases in Q_L (see equation (3.3.1)).

First we give a brief explanation of the leader's strategies in both cases. Figure 3.1 serves as illustrative support to the explanations, and shows for which levels of demand shock X a strategy is feasible. Afterwards, each strategy will be discussed.

Entry deterrence

We define entry deterrence as a strategy where the follower invests later than the leading firm. Here, given the current level of X, a large enough investment by the leading firm leads to a follower's investment trigger $X_{F,add}(Q_L)$ that is larger than X. In order for the leader's capacity to be nonnegative, the current demand shock X has to be larger than a *lower* bound $\underline{X_{add}^{det}}$. In case $X < \underline{X_{add}^{det}}$, the demand level is too low for an investment to be profitable.

Entry accommodation

Alternatively, the leader chooses the accommodation strategy, which means that the leader and follower invest simultaneously. The accommodation strategy can only occur for small capacity investments by the leader, i.e. when $Q_L \leq \widehat{Q_{L,add}}(X)$. In terms of X, entry accommodation only occurs for high levels of demand shock X, i.e. there is a *lower* bound $\underline{X_{add}^{acc}}$ for which $X > \underline{X_{add}^{acc}}$ leads to entry accommodation.

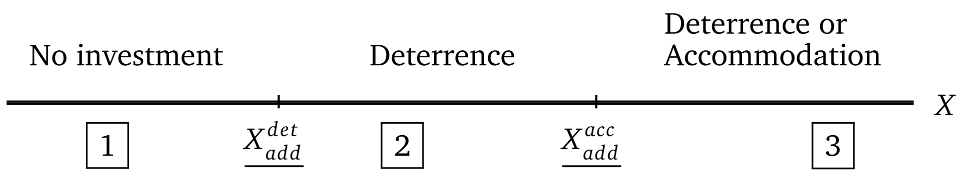


Figure 3.1: Location of accommodation and deterrence boundaries for the additive demand function.

Figure 3.1 presents the three possible regions for the additive demand function. For low values of X, investment is not optimal. Intermediate values of X enable the leader to deter entry. In the third region both strategies could be chosen. The leader will choose the strategy that maximizes its expected profit.

Let us discuss the leader's two strategies in more detail. We start with the investment decision by the leader that performs the deterrence strategy. The value function of a leader with an entry deterrence policy is given by

$$V_{L,add}^{det}(X,Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L - (\frac{\eta Q_L Q_{F,add}(Q_L)}{r})(\frac{X}{X_{F,add}(Q_L)})^{\beta}. \tag{3.3.4}$$

For the derivation of this value function, we refer to Appendix 3.B.1. Since the leading firm uses the entry deterrence policy, it incurs monopoly profits for a certain amount of time, given by the first two terms of the value function. The third term represents the investment costs necessary to install capacity of amount Q_L. However, at some point in time, the entrant will enter the market as well, which decreases the value of the leader. This is represented by the negative fourth term of the leader's value function, which describes the difference between the leader's monopoly and duopoly profits, discounted from the moment of the follower's investment $X_{F,add}(Q_L)$ back to X by a discount factor equal to $(\frac{X}{X_{F,add}(Q_L)})^{\beta}$.

Given that the leading firm uses the deterrence strategy, it will maximize its value (given by equation (3.3.4)) with respect to timing ($X_{L,add}^{det}$) and capacity size ($Q_{L,add}^{det}$). Proposition 3.2 states the optimal investment decision of the leader when it uses the entry deterrence policy.

Proposition 3.2

Suppose the additive inverse demand function $P_{t,add} = X_t - \eta Q_t$. In terms of the demand shock parameter X, the leader will consider the entry deterrence strategy whenever the current level of X lies within the interval $(\underline{X_{add}^{det}}, \infty)$, where $\underline{X_{add}^{det}}$ is implicitly given by the solution of the following equation:

$$\frac{\underline{X_{add}^{det}}}{r-\mu} - \delta_1 - \frac{\delta_1}{\beta-2}\left(\frac{\underline{X_{add}^{det}}(\beta-2)}{\delta_1\beta(r-\mu)}\right)^{\beta} = 0.$$

The value function for the leader's entry deterrence strategy, when the leader invests at X, $V_{L,add}^{det}(X)$, is equal to

$$V_{L,add}^{det}(X) = \frac{XQ_{L,add}^{det}(X)}{r-\mu} - \frac{\eta(Q_{L,add}^{det}(X))^2}{r} - \delta_1 Q_{L,add}^{det}(X) -$$

$$\left(\frac{Q^{det}_{L,add}(X)(\delta_1 r + Q^{det}_{L,add}(X)\eta)}{r(\beta-2)}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r + Q^{det}_{L,add}(X)\eta)(r-\mu)}\right)^{\beta}.$$

The optimal investment threshold $X^{det}_{L,add}$ and the corresponding capacity $Q^{det}_{L,add}$ are given by

$$X^{det}_{L,add} = \frac{\delta_1\beta(r-\mu)}{\beta-2},$$

$$Q^{det}_{L,add} = \frac{\delta_1 r}{(\beta-2)\eta}.$$

The alternative for the leader is to use the entry accommodation policy, where it is profitable for the entrant to immediately invest once the leader has invested. The value function of the leader, using the accommodation policy, $V^{acc}_{L,add}$, is equal to

$$V^{acc}_{L,add}(X,Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta((Q_L)^2 + Q_L Q_F)}{r} - \delta_1 Q_L.$$

The first two terms represent the leader's expected discounted duopoly profit. Notice that the leader will not obtain any monopoly profit, when choosing the accommodation strategy. The third term in this expression is the investment costs resulting from investment in capacity amount equal to Q_L. Proposition 3.3 gives the investment decision of the leading firm when using the entry accommodation policy.

Proposition 3.3
Consider the additive inverse demand function $P_{t,add} = X_t - \eta Q_t$. The leader can choose the entry accommodation strategy whenever the current level of X is larger than or equal to $\underline{X^{acc}_{add}}$, where

$$\underline{X^{acc}_{add}} = \frac{\delta_1\beta(r-\mu)}{\beta-4}.$$

The leader's value of the entry accommodation strategy, when investment takes place at X, is equal to:

$$V^{acc}_{L,add}(X) = \frac{r(X-\delta_1(r-\mu))^2}{8\eta(r-\mu)^2}.$$

The optimal investment threshold and corresponding capacity level for the entry accommodation strategy are given by

$$X^{acc}_{L,add} = \frac{\delta_1 \beta (r-\mu)}{\beta - 2},$$

$$Q^{acc}_{L,add} = \frac{\delta_1 r}{(\beta - 2)\eta}.$$

In case $\beta > 4$, it holds that $\underline{X^{acc}_{add}} > X^{acc}_{L,add}$, therefore the leader will invest at the accommodation lower bound $\underline{X^{acc}_{add}}$ (under the condition that it obtains a higher value from accommodation strategy compared to the deterrence strategy). However, when $2 < \beta < 4$, the accommodation boundary $\underline{X^{acc}_{add}}$ turns out to be negative, and the accommodation strategy is never possible. In this case the leader will invest at the optimal deterrence investment threshold $X^{det}_{L,add}$ (due to assumption $\beta > 2$, i.e. $r > 2\mu + \sigma^2$). Figure 3.2 gives $Q^{det}_{L,add}$, $Q^{acc}_{L,add}$ and $\widehat{Q_{L,add}}$ as a function of the demand intercept X. This figure and Figure 3.1 illustrate that for $X > \underline{X^{acc}_{add}}$ and for $X > \underline{X^{det}_{add}}$, both strategies are possible for the leader. However extensive numerical results show that the leader always chooses entry deterrence over entry accommodation, because it leads to a higher value. Thus, the accommodation strategy shall never occur under the assumption of additive demand.

Endogenous firm roles

In the first part of this section we analyzed the investment decisions of the two firms under the assumption of exogenous firm roles, i.e. both firms have information about which will be the leader and which will be the follower. In reality, the market position of two firms is not predefined. The previous results are however necessary for the analysis of endogenous firm roles, which we will discuss in the remaining part of this chapter. Under endogenous firm roles both firms have the opportunity to become the market leader, where the advantage of being the leader is that the firm can enjoy a period of monopoly profits. Once it is known which of the two firms invests first, the other firm becomes the follower. After investment by the leader, the follower acts as if the market positions are exogenously determined, since there are no strategic aspects related to its investment decision anymore. Therefore, for the investment decision of the follower, in case of endogenous firm roles, we can refer to Proposition 3.1.

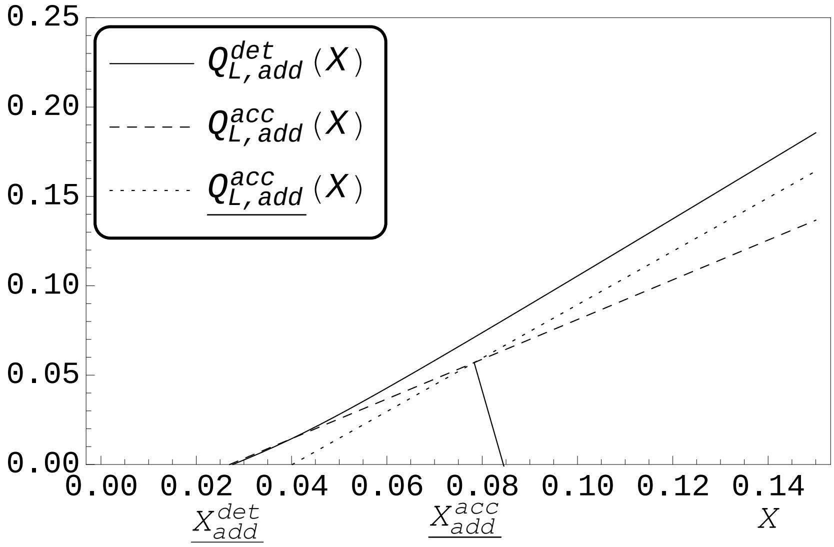


Figure 3.2: $Q^{det}_{L,add}(X)$, $Q^{acc}_{L,add}(X)$ and $\widehat{Q_{L,add}}(X)$ as a function of X. Parameter values: r =0.1, μ =0.01, σ =0.05, η =0.5, δ_1 =0.3, and δ_0 =0.

For any value of the demand intercept at which the leader's payoff exceeds the one of the follower, the firms try to preempt each other. As a result, investment will take place at a preemption trigger X_P, i.e. the moment where a firm is indifferent between waiting with investment until X reaches the follower trigger, and investing immediately. Among others, Huisman (2000) shows that the preemption trigger can be obtained by solving the following equation for X_P:

$$V^{det}_{L,add}(X_P, Q_L(X_P)) = V_{F,add}(X_P, Q_L(X_P)). \tag{3.3.5}$$

A firm does not want to invest at $X < X_P$ because in that case it would be more profitable to invest at the follower trigger. If $X > X_P$, it is more profitable for a firm to invest immediately and become the leader, than wait with investment. However, this is the case for both firms. Assume that firm 1 wants to invest at level X. Then firm 2 would preempt firm 1 and invest at $X - \epsilon$. The reaction of firm 1 is to invest even earlier, at $X - 2\epsilon$. This preemption mechanism proceeds until $X - n\epsilon = X_P$, where one of the firms will invest. Because the firms are symmetric, both have equal probabilities to become the market leader at the preemption trigger. Note that, until the preemption moment it was not yet clear for any of the two firms which would preempt the other firm, or phrased differently, which would become the leader that invests at preemption trigger X_P, and which would become the follower, that invests at investment trigger X_F.

In case of exogenous firm roles, it can easily be shown that the follower optimally invests in a larger capacity than the leader[5]:

$$Q^{det}_{L,add} = \frac{\delta_1 r}{(\beta-2)\eta} = \frac{\delta_1 r(\beta-2)}{(\beta-2)^2\eta} < \frac{\delta_1 r(\beta-1)}{(\beta-2)^2\eta} = Q^{det}_{F,add}.$$

Since the follower invests after the leader, the market share has grown to a larger level, and the follower is able to optimally invest in a larger capacity level. This holds because of the property of the additive demand function that a larger price intercept (X_t) directly results in a larger current market size. Assuming endogenous firm roles, this result will even be strengthened, because the leader will invest sooner at its preemption trigger than in case of exogenous firm roles (Huisman (2000)). This implies that the investment trigger as well as the optimal capacity of the leader are smaller than the trigger and capacity of the follower.

Result 3.1
In case of additive demand, the follower invests in a larger capacity than the leader.

Even though the majority of empirical literature states that there is a negative relationship between market entry and market share (see Kalyanaram et al. (1995)), there are examples where a later entrant takes the market. For example, EMI developed the first CT scanner, but lost in the market place because the firm lacked a technological infrastructure and marketing base in the medical field. In other instances late movers have been successful largely because they were able to exploit existing assets in areas such as marketing, distribution, and customers' reputation, e.g. IBM in personal computers and Matsushita in VCRs (Lieberman and Montgomery (1988)). A more recent example can be given in the online shopping market, where Zalando became the market leader in online shoe sales in France one year after entry, due to a huge and effective marketing campaign.

[5]Notice that the difference in capacity size only depends on β, and is thereby not affected by the investment cost δ_1 and the substitutability parameter η.

Regarding the two strategies of the leader (i.e. entry accommodation and entry deterrence), extensive numerical experiments all lead to the result that entry deterrence is more profitable than entry accommodation under endogenous firm roles. This result is irrespective of the choice of demand structure, in the next subsection we also find that under the assumption of the multiplicative demand structure this result occurs.

Multiplicative Demand 3.3.2

In this section we consider the multiplicative inverse demand function, defined by equation (3.2.1). This case has been elaborately investigated by Huisman and Kort (2014). We refer to their paper for the derivation of the corresponding results.

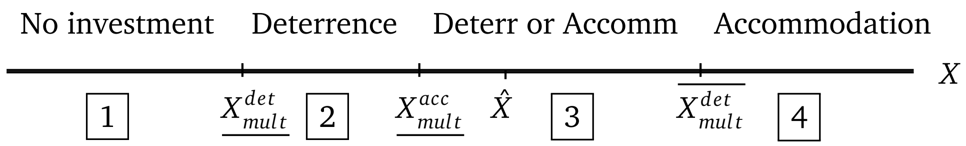


Figure 3.3: Location of accommodation and deterrence boundaries for the multiplicative demand function.

Similar to the analysis of additive demand, we first consider **exogenous firm roles**, where it is known beforehand which firm will become the leader and which one the follower. Figure 3.3 illustrates that, in contrast to the additive demand model, a multiplicative demand function results in an upper bound $\overline{X_{mult}^{det}}$ on X for the deterrence strategy[6]. Namely, entry deterrence occurs when, given a current level of X, a large enough investment by the leading firm leads to a follower's investment trigger $X_{F,mult}(Q_L)$ that is larger than X. A *lower* bound $\widehat{Q_{L,mult}}$ can be derived, for which it holds that when $Q_{L,mult} > \widehat{Q_{L,mult}}$ results in the entry deterrence strategy. Translating this in terms of X, entry deterrence can only occur when the value of demand shock X is below an

[6]For the additive demand function, we find that there is no upper bound $\overline{X_{add}^{det}}$ on X for the deterrence strategy (see proof of Proposition 3.2). In fact, for all X, it holds that the optimal capacity level $Q_{L,add}^{det}(X)$ is such that $Q_{L,add}^{det}(X) > \widehat{Q_{L,add}}$. This is intuitive since the additive inverse demand function, given by equation (3.2.2), is not restricted by a fixed price intercept. This implies that for $X > \underline{X_{add}^{det}}$ it is always optimal for the leader to place a large enough capacity in the market to deter the entrant.

upper bound $\overline{X^{det}_{mult}}$, i.e. $X < \overline{X^{det}_{mult}}$. Otherwise, the market is large enough, that it is optimal for the follower to enter immediately once the leader has invested. Note that, compared to additive demand, it is less worthwhile in this case to wait and invest in a larger capacity for high X, because the upper bound of capacity does not increase with X. That is, the multiplicative demand structure has an upper bound on the total market size. Therefore, compared to an additive demand model, entry deterrence is more difficult to achieve under multiplicative demand.

Figure 3.3 shows that there are four possible regions for a multiplicative demand function, where in the third region both strategies are possible. In this region, the leader will choose the strategy that maximizes its expected profit. For some threshold $\hat{X}$, the leader is indifferent between the deterrence and the accommodation strategies, i.e.

$$\hat{X} = min\{X \in (\underline{X^{acc}_{mult}}, \overline{X^{det}_{mult}}) | V^{det}_{L,mult}(X) = V^{acc}_{L,mult}(X)\}.$$

When $X < \hat{X}$, it is optimal for the leader to deter entry in the third region, and when $X > \hat{X}$ entry accommodation will occur.

In case of exogenous firm roles, it can easily be shown that the leader turns out to be the firm with the larger capacity[7]:

$$Q^{det}_{L,mult} = \frac{1}{(\beta+1)\eta} = \frac{\beta+1}{(\beta+1)^2\eta} > \frac{\beta}{(\beta+1)^2\eta} = Q^{det}_{F,mult}.$$

Under the assumption of **endogenous firm roles**, Huisman and Kort (2014) show that the leader will only have the largest capacity when uncertainty is sufficiently large (see also Figure 3.4). These roles explain that the preemption threat forces the leader to invest early, i.e. the market will be very small at the moment of investment (i.e. at the preemption trigger). However, a high level of uncertainty generates the value of waiting, which in turn delays the moment of preemption and therefore also the moment of investment by the leader. This enables the leader to invest in a larger capacity.

Furthermore, as was explained for additive inverse demand, extensive numerical experiments reveal that the entry deterrence strategy will be more

[7]Notice that, also for multiplicative demand, the difference in capacity size only depends on β.

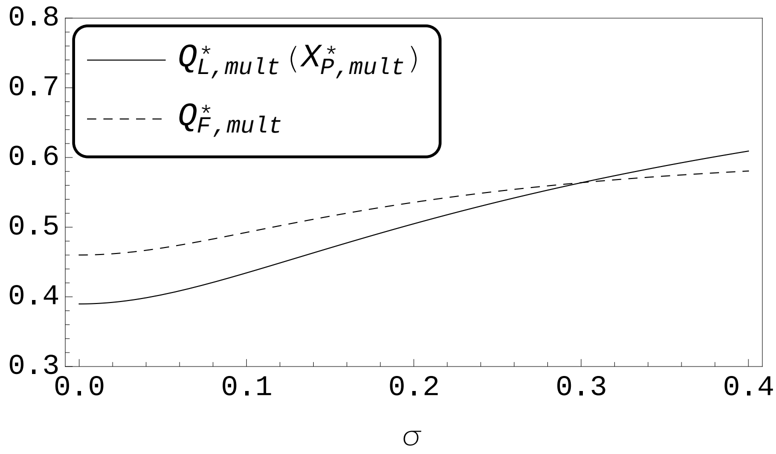


Figure 3.4: The optimal investment capacities for the leader and the follower, as a function of uncertainty σ, in case of multiplicative demand function. Parameter values: r =0.1, μ =0.04, η =0.5, δ_1 =1, and δ_0 =0.

profitable for the leader under endogenous firm roles. As a result, simultaneous investment will not occur, when the firms play the preemption game.

3.4 Fixed Investment Costs and the Effect of the Cost Structure

In this section, we include a fixed term to the investment costs and assume that investment costs are equal to $\delta_1 Q_t + \delta_0$. Including constant investment costs allows us to take into account the iso-elastic demand model. The analysis of iso-elastic demand is similar to the analysis of the additive demand case in Section 3.3. The formulas of the different models that are used to derive the results in this section, can be found in Appendix 3.B.2.

There are two main effects that explain how increasing costs affect the firm's optimal choice of capacity and investment timing. There is a direct effect present in the sense that higher investment costs make investment more expensive and will therefore lead to a smaller optimal capacity size. On the other hand, higher investment costs result in a delay in the optimal moment of investment, which, in turn, leads to a larger optimal capacity (which is the indirect effect). Which of these effects dominate, determines whether a firm will invest in a larger or a smaller capacity amount for increasing investment costs. However, considering the timing of investment, the firm always delays the moment of investment when a higher level of investment cost is considered, no matter what the choice of demand structure is.

In the following subsections we focus on the fixed and variable cost effect on the capacity size and investment timing of the two duopoly firms. In summary, we find that for all demand structures a higher constant cost (i.e. higher δ_0) leads to a higher investment trigger and a corresponding higher capacity level (i.e. the indirect effect dominates). This result is empirically supported by Bai et al. (2008) who lend support to the hypothesis that investment in capacity is positively associated with the proportion of fixed costs in the cost structure[8].

However, the demand structures give conflicting results considering the effect of a higher variable cost. Analyzing the firm's optimal investment decisions under additive demand, a firm will delay its investment decision when uncertainty increases, which in turn leads to a larger capacity choice (referred to as indirect effect beforehand). However, when the iso-elastic or multiplicative demand is considered, the direct effect dominates, and we observe a decreasing pattern in optimal capacity for an increase in variable costs. The difference in results between additive versus multiplicative and iso-elastic demand can be retrieved from their corresponding structure of inverse demand. In case of additive demand, a higher level of X corresponds to a larger total market size. Therefore, a late mover has an extra incentive to wait for a higher level of X when variable costs are high. Due to the increased market size, the second entrant can also make a larger capacity investment. For the other two demand functions, an increase in X only results in an increase in price, since X is a multiplication with the 'demand part' of the price function. By understanding how a firm makes its investment decision with a specific form of demand, one could also apply the results to other types of demand functions that are not examined in this chapter.

3.4.1 Additive Demand

Figure 3.5 illustrates how sensitive the investment decisions of the leader and follower are with respect to the two cost parameters δ_1 and δ_0. One can see

[8] This paper empirically tests this prediction using 6000 department level observations from California hospitals for the period 1998-2005.

that for higher constant costs (i.e. higher δ_0) the indirect effect dominates. Here, a firm will delay investment and consequently invest in a larger capacity amount, when facing a higher fixed cost. The same happens for higher linear costs (i.e. higher δ_1). As discussed earlier in this section, this latter result is due to the structure of the additive demand, which allows for an increase in market size, for an increase in uncertainty parameter X. This result also partly relates to empirical research from Lieberman (1987), who examines the increase in new chemical plants over the period from the late 1950 through the early 1980s. Lieberman (1987) reveals that each 1% differential in unit production cost between small and large scale plants leads to about a 2% increase in the average size of new plants. His research departs from our findings with additive demand, when he shows that entrants build smaller capacities than incumbents, where we can conclude that including constant investment costs does not change the result of Section 3.3.1, that in case of the additive demand function the follower invests in a larger capacity than the leader.

Iso-elastic Demand 3.4.2

The iso-elastic inverse demand function, given by equation (3.2.3), does not impose an upper bound on the total market size. Since the follower invests later than the leader, the follower makes a larger capacity investment than the leader, just like the case of the additive demand function. In general, when market size is not restricted, the follower will invest in a larger capacity than the leader. An example of a restricted market size (multiplicative demand) will be discussed in Section 3.4.3. This knowledge can be used for demand functions not examined in this chapter.

Figure 3.6 illustrates the sensitivity of the optimal investment decisions of two firms with respect to the two investment cost parameters. An increase of the constant investment cost parameter (i.e. δ_0) will lead to a similar result as was observed under the assumption of additive demand, i.e the indirect effect dominates. Higher constant costs delay investment, and therefore lead (both firms) to a larger capacity investment. The effect of the linear cost parameter however, will delay investment but results in a smaller capacity investment. Here, the direct effect of increasing costs related to the optimal capacity size dominates the indirect effect. Figure 3.6 illustrates this for a specific numerical

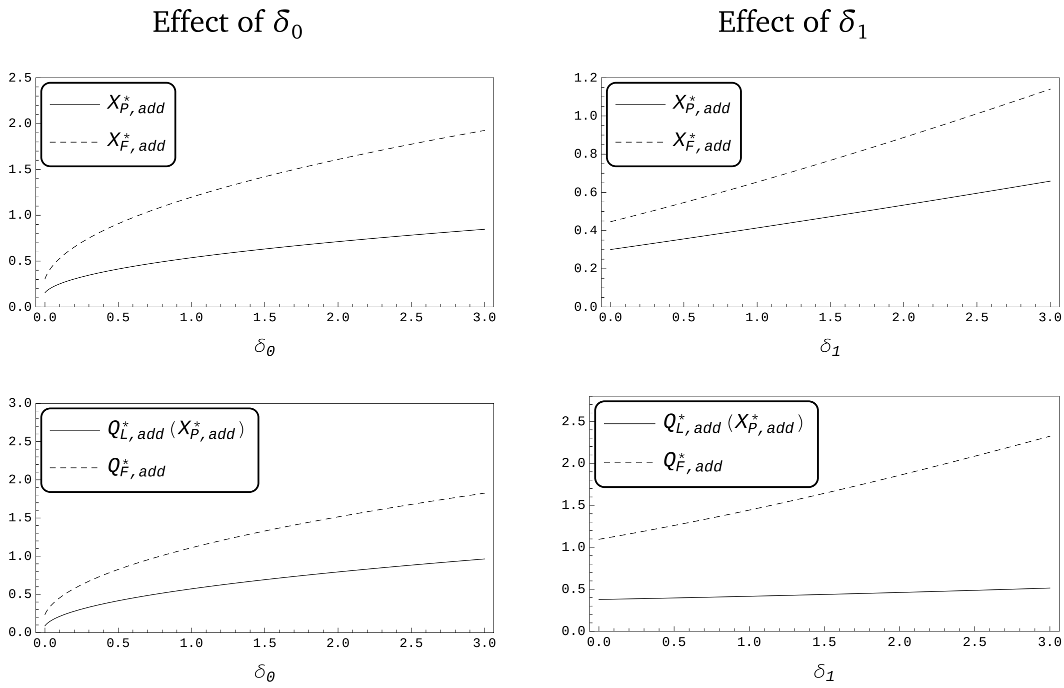


Figure 3.5: The optimal investment trigger and optimal capacities for the leader and the follower, as a function of cost parameters δ_1 and δ_0, when demand is additive. Parameter values: $r=0.1$, $\mu=0.02$, $\sigma=0.1$, $\eta=0.5$, and $\delta_1=1$ in the two left-hand graphs, $\delta_0=0.5$ in the two right-hand graphs.

example. We show the robustness of this result in the sense that extensive numerical experiments with different parameter values lead to the same result (see Appendix 3.C).

3.4.3 Multiplicative Demand

Regarding the effect of increasing the constant cost parameter, the result that the indirect effect dominates the direct effect holds for most cases, and also for the multiplicative demand function, as illustrated in Figure 3.7. However, regarding the capacity size of the leader and follower, the result for the multiplicative demand function is different than in the other two cases. Here the leader invests for most cases in a larger capacity than the follower. Kalyanaram et al. (1995) states that one of the emerging generalizations found in the empirical

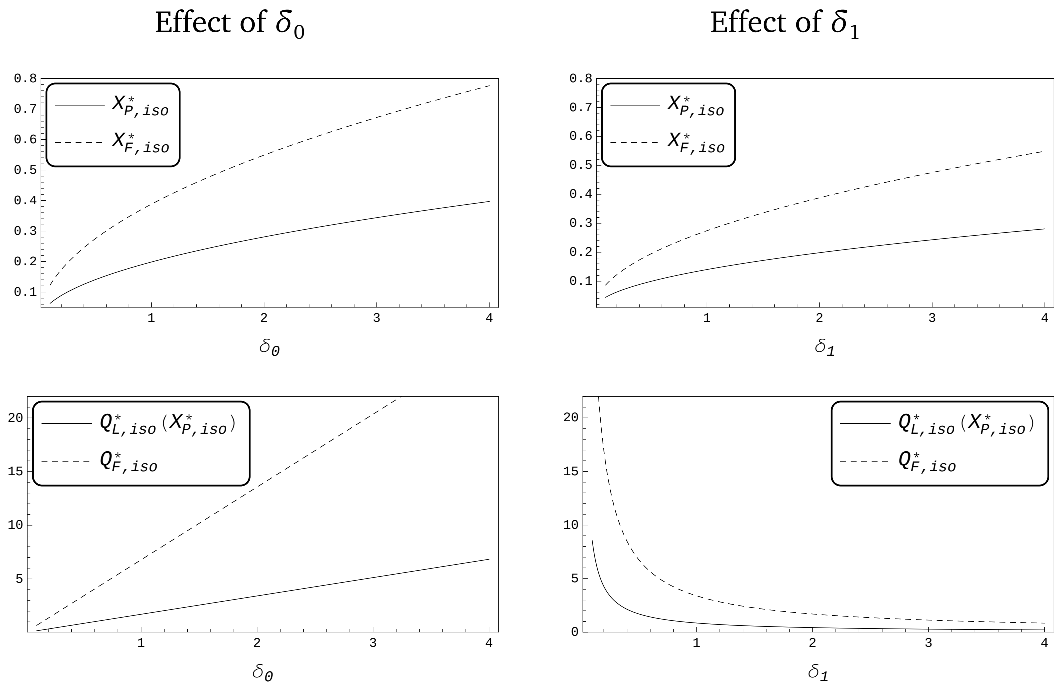


Figure 3.6: The optimal investment trigger and optimal capacities for the leader and the follower, as function of cost parameters δ_1 and δ_0, when demand is iso-elastic. Parameter values: r =0.1, μ =0.02, σ =0.1, η =0.5, and δ_1 =1 in the two left-hand graphs, δ_0 =0.5 in the two right-hand graphs.

literature is that there is a negative relationship between the order of market entry and market share. Other papers that support this generalization are Uran et al. (1986) and Kalyanaram and Wittink (1994). However, this chapter finds that in a theoretical framework, this result is only valid with the multiplicative demand. One has to be aware that by choosing the demand function, a specific assumption is made about the (increase) in market size. Therefore a thoughtful decision about the structure of inverse demand has to be made.

In Section 3.3.2 we explained that the use of the multiplicative demand function implies an upper bound on quantity, being independent of X, in order to guarantee a positive output price. Since the leader chooses its capacity amount first, it is able to take a greater share of the market. This effect dominates especially in case of high constant investment costs. For relatively low constant investment costs, the linear investment cost part dominates the total investment

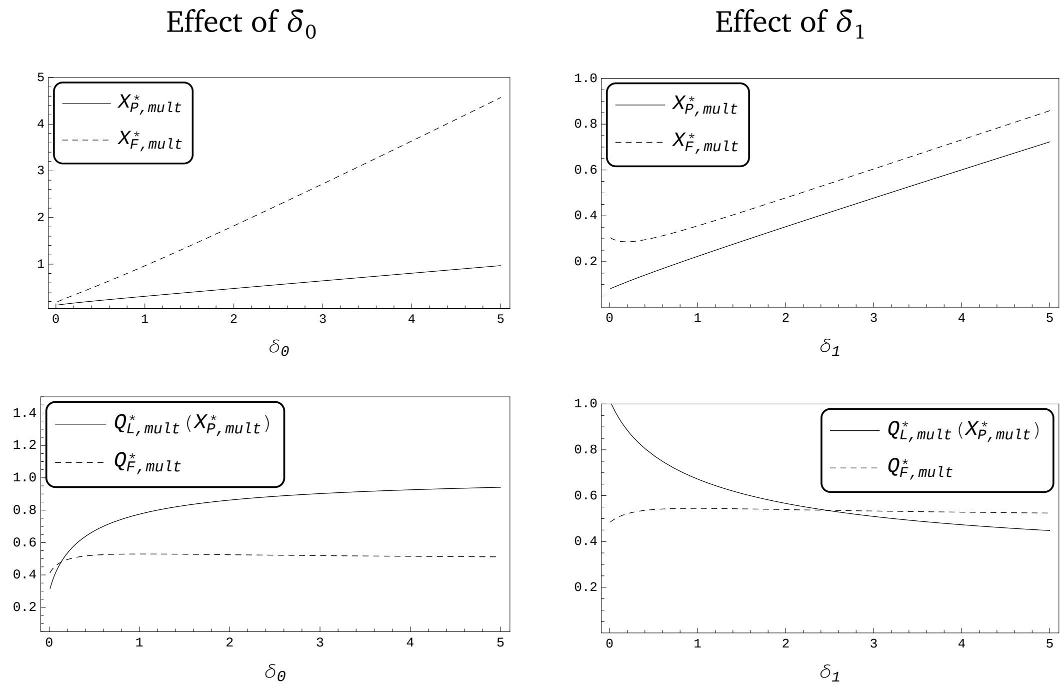


Figure 3.7: The optimal investment trigger and optimal capacities for the leader and the follower, as function of cost parameters δ_1 and δ_0, when demand is multiplicative. Parameter values: r =0.1, μ =0.02, σ =0.1, γ =0.5, and δ_1 =1 in the two left-hand graphs, δ_0 =0.5 in the two right-hand graphs.

costs. This implies that the leader, that aims to preempt the follower, will invest soon and in a small capacity amount. The follower, however, invests at a later point which enables it to invest in a larger capacity investment than the leader.

The right-hand graphs in Figure 3.7 illustrate the effect of increasing linear investment costs on the investment decisions of the two firms. The lower right-hand graph illustrates the tradeoff between constant and linear part of the total investment costs. When the linear part of the investment costs dominate the total costs, the follower will undertake a larger capacity investment than the leader, and when the constant part of the investment costs dominates, the opposite result occurs. This effect is stated in Result 3.2.

We can observe a third effect (See right-hand graphs of Figure 3.7) that is strongest for very low δ_1, namely a strategic effect. For very low linear investment costs, the optimal investment threshold of the follower is decreasing in linear costs. The direct effect explains that for a slightly higher δ_1 the leader invests in a smaller optimal capacity. This gives the follower a better investment incentive, therefore it will invest earlier in a larger capacity. However, for a larger δ_1, the cost effect dominates the strategic effect, and both firms will invest later in a smaller capacity.

Result 3.2

For the multiplicative demand function, the effect of cost parameters δ_0 and δ_1 determines which firm will have the larger capacity in the market. When the linear costs dominate the total cost function ($\delta_1 > \delta_1^{-1}(\delta_0)$), the corresponding low optimal leader capacity makes the follower the larger firm in the market. When the constant costs dominate the total cost function ($\delta_0 > \delta_0(\delta_1)$), the corresponding high optimal leader capacity forces the follower to invest in a small capacity, which makes the follower the smaller firm in the market. $\delta_0(\delta_1)$ ($\delta_1^{-1}(\delta_0)$) denotes the level of δ_0 (δ_1) for which the leader and the follower invest in the same capacity size. We define $\delta_0(\delta_1)$ by:

$$\delta_0(\delta_1) = \min\{\delta_0 | Q_{L,det}^{mult}(\delta_0, \delta_1) = Q_{F,det}^{mult}(\delta_0, \delta_1), \delta_1\}.$$

Result 3.2 is illustrated in Figure 3.8.

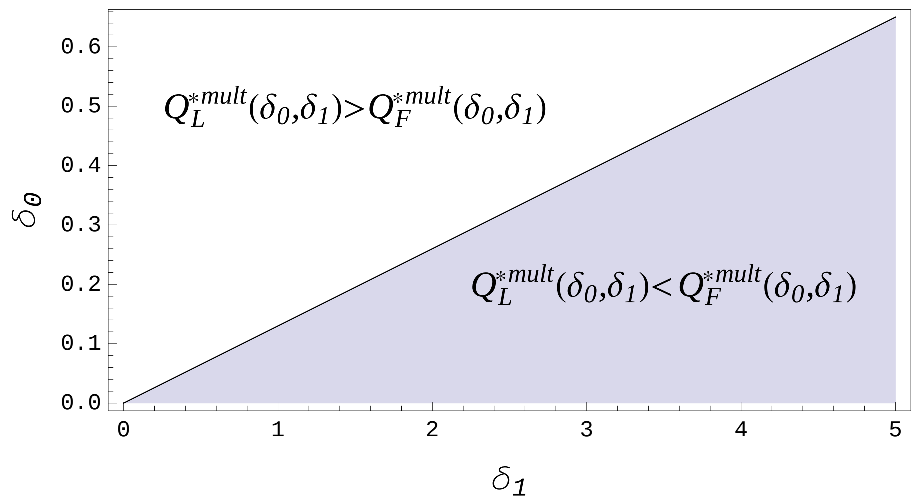


Figure 3.8: The relationship between the optimal quantity of the leader (Q_L^{*mult}) and follower (Q_F^{*mult}) in case of multiplicative demand. Parameter values: r =0.1, μ =0.02, σ =0.1 and η =0.5.

3.5 Conclusion

This chapter studies the effect of three commonly used demand functions, additive, multiplicative, and iso-elastic demand, on optimal investment decisions in a duopoly. We analyze the optimal timing as well as the optimal size of investment of two firms in the corresponding markets in a real options context.

Essentially, the leader can use two strategies. The deterrence strategy gives a period of monopoly profits before the entry of the follower, and can be achieved by overinvestment. On the other hand, the accommodation strategy implies that the follower invests at the same time as the leader. This will only be the leader's strategy when overinvestment is too expensive. We find that for the additive demand function, the leader will always choose the deterrence strategy, when exogenous firm roles are considered.

Furthermore, we find that for most market situations the result that the second investor invests in a larger capacity amount than the first investor, is robust concerning the choice of demand model. The opposite result holds however for the multiplicative demand function when uncertainty and constant costs are sufficiently high. For high uncertainty, there is so much value in waiting that even in the preemption game the first investor invests so late that it can make a larger capacity investment. Also, high constant costs correspond to a large capacity investment by the leader.

Furthermore, we find that the effect of an increase in variable investment costs can be explained by a direct and an indirect effect. A slightly higher variable cost decreases the optimal capacity (direct effect), but also delays investment which corresponds to a larger capacity amount (indirect effect). In case of the additive demand the indirect effect dominates, i.e. larger linear costs lead to a larger capacity choice. This is contrary to the multiplicative and iso-elastic demand function, where higher linear costs result in a later investment but smaller capacity. i.e. the direct effect dominates.

We made the assumption of a lump-sum investment game. A possible interesting extension of the current work might be to investigate a firm's optimal investment decisions when the leader can gradually extend its initial capacity when the price moves up. Another extension would be to consider an n-firm oligopoly framework, and investigate if the duopoly results also hold for a market with more firms. A useful starting point may be the dynamic oligopoly model of Bouis et al. (2009), by including optimal capacity. In addition, it would be interesting to analyse how our results change when product differentiation is considered.

3.A Proof of Propositions

3.A.1 Proof of Proposition 3.1

The follower's profit in a model with additive demand and linear costs is equal to

$$\pi_{t,F,add} = (X_t - \eta(Q_{t,F} + Q_{t,L}))Q_{t,F}.$$

$V_{F,add}(X, Q_F, Q_L)$ denotes the follower's discounted expected value when it invests in a capacity amount of Q_F. It is equal to:

$$V_{F,add}(X, Q_F, Q_L) = \mathbb{E}\left(\int_{t=0}^{\infty} e^{-rt}(X_t - \eta(Q_{t,F} + Q_{t,L}))Q_{t,F}\,\mathrm{d}t - \delta_1 Q_F\right) \tag{3.A.1}$$

$$= \frac{XQ_F}{r-\mu} - \eta\frac{Q_F^2 + Q_L Q_F}{r} - \delta_1 Q_F. \tag{3.A.2}$$

Differentiating (3.A.1) with respect to Q_F results into the follower's optimal capacity for a given X and Q_L:

$$Q_{F,add}(X, Q_L) = \frac{Xr - (r-\mu)(r\delta_1 + Q_L\eta)}{2\eta(r-\mu)}. \tag{3.A.3}$$

Substitution of (3.A.3) into (3.A.1) gives the value function of the follower after investment:

$$V_{F,add}(X, Q_L) = \frac{(Xr - (r-\mu)(r\delta_1 + Q_L\eta))^2}{4r\eta(r-\mu)^2}.$$

The value of the firm before investment, i.e. the value of waiting, takes the form $f(X) = AX^{\beta}$, where A is a constant to be determined and β is the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0.$$

Denote with $X_F(Q_F)$ the investment moment of the follower. Value matching and smooth pasting the follower's value of waiting with the value function after investment results in solving the following equations:

$$A_{F,add}X_{F,add}^{\beta} = V_F(X_{F,add}(Q_L),Q_L),$$

$$\beta A_{F,add}X_{F,add}^{\beta-1} = \frac{\partial V_{F,add}(X_{F,add}(Q_L),Q_L)}{\partial X}.$$

Therefore,

$$A_{F,add}(Q_L) = \left(\frac{r(\beta-2)}{\beta(r\delta_1+Q_L\eta)(r-\mu)}\right)^{\beta}\left(\frac{(r\delta_1+Q_L\eta)^2}{(\beta-2)^2 r\eta}\right),$$

$$X_{F,add}(Q_L) = \frac{\beta(r\delta_1+Q_L\eta)(r-\mu)}{r(\beta-2)}, \tag{3.A.4}$$

so that the follower's optimal capacity is equal to

$$Q_{F,add}(Q_L) = Q_{F,add}(X_{F,add}(Q_L),Q_L) = \frac{r\delta_1+Q_L\eta}{r(\beta-2)}. \tag{3.A.5}$$

□

Proof of Proposition 3.2 3.A.2

The value function after investment by the leader that uses the deterrence strategy is equal to

$$\begin{aligned} V_{L,add}^{det}(X,Q_L) = {} & \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L \\ & - \left(\frac{\eta Q_L Q_{F,add}(Q_L)}{r}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r+Q_L\eta)(r-\mu)}\right)^{\beta}. \end{aligned} \tag{3.A.6}$$

Substitution of (3.A.5) into (3.A.6) gives

$$\begin{aligned} V_{L,add}^{det}(X,Q_L) = {} & \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L \\ & - \left(\frac{Q_L(\delta_1 r+Q_L\eta)}{r(\beta-2)}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r+Q_L\eta)(r-\mu)}\right)^{\beta}. \end{aligned}$$

In order to find the optimal leaders capacity Q_L^{det}, we need to take the first order derivative of this value with respect to Q_L, and set it equal to zero, which results in the following condition:

$$\frac{\partial V_{L,add}^{det}(X,Q_L)}{\partial Q_L} = \left(\frac{Q_L(\beta-2)\eta - r\delta_1}{r(\beta-2)}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r + Q_L\eta)(r-\mu)}\right)^{\beta} + \frac{X}{r-\mu} - \frac{2Q_L\eta}{r} - \delta_1 = 0 \tag{3.A.7}$$

Using the envelope theorem gives that

$$\frac{dV_{L,add}^{det}(X,Q_L(X))}{dX} = \frac{\partial V_{L,add}^{det}(X,Q_L)}{\partial X} + \frac{\partial V_{L,add}^{det}(X,Q_L)}{\partial Q_L}\frac{\partial Q_L}{\partial X} = \frac{\partial V_{L,add}^{det}(X,Q_L)}{\partial X},$$

since $\frac{\partial V_L(X,Q_L)}{\partial Q_L}$ is zero, as is shown in condition (3.A.7). Therefore the value matching and smooth pasting conditions are given by

$$AX^{\beta} = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L - \left(\frac{Q_L(\delta_1 r + Q_L\eta)}{r(\beta-2)}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r + Q_L\eta)(r-\mu)}\right)^{\beta},$$

$$\beta AX^{\beta-1} = \frac{Q_L}{r-\mu} - \frac{Q_L}{r-\mu}\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r + Q_L\eta)(r-\mu)}\right)^{\beta-1},$$

and result in the following leader's investment threshold

$$X_{L,add}^{det}(Q_L) = \frac{\beta(r\delta_1 + Q_L\eta)(r-\mu)}{r(\beta-1)}. \tag{3.A.8}$$

Solving (3.A.8) simultaneously with (3.A.7), results in the optimal leader's investment trigger $X_{L,add}^{det}$, with corresponding optimal capacity $Q_{L,add}^{det}$:

$$X_{L,add}^{det} = \frac{\delta_1\beta(r-\mu)}{\beta-2},$$

$$Q_{L,add}^{det} = \frac{\delta_1 r}{(\beta-2)\eta}.$$

To find the boundaries on X, i.e. lower bound $\underline{X_{add}^{det}}$ and upper bound $\overline{X_{add}^{det}}$, we have to substitute $Q_L = 0$ and $Q_L = \widehat{Q_{L,add}}$, respectively, into condition (3.A.7), and solve for X. $\widehat{Q_{L,add}}$ is defined by equation (3.3.3) and can be found by solving $X_{F,add}(Q_L) = 0$. Substitution of $Q_L = 0$ into equation (3.A.7) results in:

$$\psi(X) = \frac{X}{r-\mu} - \delta_1 - \frac{\delta_1}{\beta-2}\left(\frac{X(\beta-2)}{\delta_1\beta(r-\mu)}\right)^{\beta} = 0.$$

Since

$$\begin{aligned}
\psi(0) = &\quad -\delta_1 < 0, \\
\psi(X_{F,add}(0)) = &\quad \frac{\delta_1}{\beta-2} > 0, \\
\tfrac{\partial\psi(X)}{\partial X} = &\quad \frac{1}{r-\mu}(1-(\frac{(\beta-2)X}{\delta_1\beta(r-\mu)})^{\beta-1}) > 0 \qquad \text{for } X \in (0, X_F(0)),
\end{aligned}$$

it holds that $\underline{X}_{add}^{det}$ exists. Furthermore $\underline{X}_{add}^{det} < \underline{X}_{add}^{acc}$, because

$$\begin{aligned}
\tfrac{\partial\psi(X)}{\partial X} &= \frac{1}{r-\mu}\left(1-\left(\frac{(\beta-2)X}{\delta_1\beta(r-\mu)}\right)^{\beta-1}\right) = 0 \quad \text{for } X = X_{F,add}(0), \\
\tfrac{\partial\psi(X)}{\partial X} &= \frac{1}{r-\mu}\left(1-\left(\frac{(\beta-2)X}{\delta_1\beta(r-\mu)}\right)^{\beta-1}\right) < 0 \quad \text{for } X \in (X_{F,add}(0), \infty), \\
\tfrac{\partial\psi(\underline{X}^{acc})}{\partial X} &= \frac{1}{r-\mu}\left(1-\left(\frac{\beta-2}{\beta-4}\right)^{\beta-1}\right) < 0,
\end{aligned}$$

which indicates that $\underline{X}_{add}^{acc} > X_F(0) > \underline{X}_{add}^{det}$ for $\beta > 4$. When $\beta < 4$, $\underline{X}_{add}^{acc}$ is negative and therefore dispensable.

Next we show that $\overline{X}_{add}^{det}$ does not exist. Solving condition (3.A.7) for Q_L can lead to either a minimum or a maximum value. Substitution of $\widehat{Q_{L,add}}$ into (3.A.7) leads to a unique value $\bar{X} = \frac{\delta_1\beta(r-\mu)}{2(\beta-2)}$ that makes $\widehat{Q_{L,add}}$ the capacity choice that corresponds to a *minimum* leader's value. Solving condition (3.A.7) numerically for the $Q_L(X)$ that leads to a maximum leader's value, gives as result that the optimal capacity will always be bigger than the minimum boundary for Q_L, i.e. $Q_L(X) > \widehat{Q_{L,add}} \quad \forall X$. Therefore, there is no upper bound for the deterrence strategy, considering the additive demand function. □

Proof of Proposition 3.3 3.A.3

The value function after investment, by the leader that uses the accommodation strategy is equal to

$$V_{L,add}^{acc}(X, Q_L, Q_F) = \frac{XQ_L}{r-\mu} - \frac{\eta((Q_L)^2 + Q_L Q_F)}{r} - \delta_1 Q_L. \tag{3.A.9}$$

Substitution of (3.A.3) into (3.A.9) and maximizing with respect to Q_L gives the optimal capacity size of the leader, as a function of X

$$Q^{acc}_{L,add}(X) = \frac{r(X-(r-\mu)\delta_1)}{2\eta(r-\mu)}. \tag{3.A.10}$$

Substitution of (3.A.10) and (3.A.3) into (3.A.9) leads to

$$V^{acc}_{L,add}(X) = \frac{r(X-(r-\mu)\delta_1)^2}{8\eta(r-\mu)}. \tag{3.A.11}$$

Solving for the value matching and smooth pasting conditions leads to the following optimal investment trigger and the corresponding leader's capacity level:

$$X^{acc}_{L,add} = \frac{\delta_1\beta(r-\mu)}{\beta-2},$$

$$Q^{acc}_{L,add} = \frac{\delta_1 r}{(\beta-2)\eta}. \tag{3.A.12}$$

The accommodation strategy can only occur for level of $X > \underline{X}^{acc}_{add}$. The leader can only consider the accommodation strategy if the optimal leader's capacity $Q^{acc}_{L,add}$ leads to immediate investment of the follower, i.e. for $\underline{X}^{acc}_{add}$ it should hold that

$$X_{F,add}(Q^{acc}_{L,add}(\underline{X}^{acc}_{add})) \leq \underline{X}^{acc}_{add}. \tag{3.A.13}$$

Substitution of (3.A.12), (3.A.4) into (3.A.13) gives

$$\underline{X}^{acc}_{add} = \frac{\delta_1\beta(r-\mu)}{\beta-4}.$$

□

Additional Proofs 3.B

Derivation of the Leader's Deterrence Value Function 3.B.1

Before the entry of the follower, the leader has a monopoly position. After entry of the follower, the monopoly turns into a duopoly. In the following we derive the value function of the leader. The value function of the leader when the follower has not yet entered the market is equal to

$$V_{L,add}^{mon}(X,Q_L) = B_1 X^{\beta} + \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L, \tag{3.B.1}$$

where $B_1 X^{\beta}$ represents the shift in value when at some point in time the leader has to share the market with the follower. The value of the leader when it produces in a duopoly is equal to

$$V_{L,add}^{JI}(X,Q_L) = \frac{XQ_L}{r-\mu} - \frac{Q_L^2 + Q_L Q_F}{r} - \delta_1 Q_L. \tag{3.B.2}$$

At the investment threshold of the follower, it holds that $V_{L,add}^{mon}(X,Q_L) = V_{L,add}^{JI}(X,Q_L)$. Furthermore, the value function of the leader has to be smooth in the threshold (X_F). Therefore, it holds that

$$V_{L,add}^{det}(X,Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L - \left(\frac{\eta Q_L Q_{F,add}(Q_L)}{r}\right)\left(\frac{X}{X_{F,add}(Q_L)}\right)^{\beta}. \tag{3.B.3}$$

The values $Q_F(Q_L)$ and $X_F(Q_L)$ are known and can be substituted in equation (3.B.3):

$$V_L^{det}(X,Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L - \left(\frac{\eta Q_L Q_{F,add}(Q_L)}{r}\right)\left(\frac{Xr(\beta-2)}{\beta(\delta_1 r + Q_L\eta)(r-\mu)}\right)^{\beta}. \tag{3.B.4}$$

□

3.B.2 Formulas in Section 4

The solution method towards finding the optimal value and investment decisions of the follower, in case of additional constant investment costs, is similar to the case with only unit investment costs (see Appendix 3.A.1). The follower's optimal capacity and timing of investment for the additive, multiplicative and iso-elastic demand structure, resulting from the mentioned solution method, are given in the following. The leader's optimal investment decisions are found numerically. We give subsequently for each demand structure the implicit equations that have to be solved in order to find the leader's optimal capacity and timing of investment.

Additive demand.
The investment trigger and corresponding optimal capacity are equal to

$$X_{F,add}(Q_L) = \frac{r(\beta-1)(r\delta_1 + Q_L\eta)(r-\mu) + \sqrt{A_{add}}}{r^2(\beta-2)},$$

$$Q_{F,add}(Q_L) = \frac{r\delta_1 + Q_L\eta + \sqrt{A_{add}}}{2\eta(\beta-2)},$$

where

$$A_{add} = (r^2\delta_1^2 + 2r(2(\beta-2)\beta\delta_0 + Q_L\delta_1)\eta + Q_L^2\eta^2).$$

Given $X_{F,add}(Q_L)$ and $Q_{F,add}(Q_L)$, the value of the leader is equal to

$$V_{L,add}^{det}(X,Q_L) = \frac{XQ_L}{r-\mu} - \frac{\eta(Q_L)^2}{r} - \delta_1 Q_L - \delta_0 - \left(\frac{\eta Q_L Q_{F,add}(Q_L)}{r}\right)\left(\frac{X}{X_{F,add}(Q_L)}\right)^{\beta}.$$

Take the first order partial derivative of $V_{L,add}^{det}(X,Q_L)$ with respect to Q_L, and set equal to zero. Solving this equation for Q_L gives $Q_L(X)$. We set the option value of the follower equal to the value of the leader. The option value of the follower is given by:

$$V_{F,add}^{option}(X,Q_L(X)) =$$

$$\Big(\frac{Q_F(Q_L(X))X_{F,add}(Q_L(X))}{r-\mu}-\frac{\eta Q_{F,add}(Q_L(X))(Q_{F,add}(Q_L(X))+Q_L(X))}{r} \\ -\delta_1 Q_{F,add}(Q_L(X))-\delta_0\Big)\Big(\frac{X}{X_F(Q_L(X))}\Big)^{\beta}.$$

As is explained in Section 3.3.1, the leader will invest at its preemption trigger, which can be obtained by solving the following equation for X_P:

$$V_{L,add}^{det}(X_P, Q_L(X_P)) = V_{F,add}^{option}(X_P, Q_L(X_P)).$$

Substitution of X_P^* into $Q_L(X)$ gives $Q_{L,add}^*(X_P^*)$. Similar, substitution of $Q_{L,add}^*(X_P^*)$ into $X_{F,add}(Q_L)$ and $Q_{F,add}(Q_L)$ gives the optimal trigger and capacity of the follower. □

Multiplicative demand.
The investment trigger and corresponding optimal capacity of the follower are given by:

$$X_{F,mult}(Q_L) = \frac{\beta(\delta_1+2\delta_0\eta-Q_L\delta_1\eta)(r-\mu)+\sqrt{A_{mult}}}{(Q_L\eta-1)^2(\beta-1)},$$

$$Q_{F,mult}(Q_L) = \frac{(\delta_1-2\beta\delta_0\eta-Q_L\delta_1\eta)(r-\mu)+\sqrt{A_{mult}}}{2(1+\beta)\delta_1\eta(r-\mu)},$$

respectively, where

$$A_{mult} = (4\beta^2\delta_0^2\eta^2-4\beta^2\delta_0\delta_1\eta(Q_L\eta-1)+\delta_1^2(Q_L\eta-1)^2)(r-\mu)^2.$$

Given $X_{F,mult}(Q_L)$ and $Q_{F,mult}(Q_L)$, the value of the leader is the following:

$$V_{L,mult}^{det}(X,Q_L) = \frac{XQ_L(1-\eta Q_L)}{r-\mu}-\delta_1 Q_L-\delta_0 \\ -\Big(\frac{\eta Q_L Q_{F,mult}(Q_L)X_{F,mult}(Q_L)}{r-\mu}\Big)\Big(\frac{X}{X_{F,mult}(Q_L)}\Big)^{\beta}.$$

Take the first order partial derivative of $V_{L,mult}^{det}(X,Q_L)$ with respect to Q_L, and set equal to zero. Solving this equation for Q_L gives $Q_L(X)$. We value match the option value of the follower to the value of the leader. The option value of the follower is given by:

$$V_{F,mult}^{option}(X, Q_L(X)) = \Big(\frac{X_{F,mult}(Q_L(X))Q_{F,mult}(Q_L(X))(1-\eta(Q_{F,mult}(Q_L(X))+Q_L(X)))}{r-\mu} - \delta_1 Q_{F,mult}(Q_L(X)) - \delta_0\Big)\Big(\frac{X}{X_{F,mult}(Q_L(X))}\Big)^{\beta}.$$

The preemption trigger can be obtained by solving the following equation for X_P:

$$V_{L,mult}^{det}(X_P, Q_L(X_P)) = V_{F,mult}^{option}(X_P, Q_L(X_P)). \tag{3.B.5}$$

Substitution of X_P^* into $Q_L(X)$ gives $Q_{L,mult}^*(X_P^*)$. Similar, substitution of $Q_{L,mult}^*(X_P^*)$ into $X_{F,mult}(Q_L)$ and $Q_{F,mult}(Q_L)$ gives the optimal trigger and capacity of the follower. □

Iso-elastic demand.
The investment trigger and corresponding optimal capacity of the follower are equal to:

$$X_{F,iso}(Q_L) = \Big(\frac{\beta(\gamma-1)\delta_0 + Q_L(2\beta-1)\delta_1 + \sqrt{A_{iso}}}{2Q_L(\beta-1)}\Big)\Big(Q_L + \frac{\beta\delta_0(1-\gamma)+Q_L\delta_1+\sqrt{A_{iso}}}{2\delta_1(\beta\gamma-1)}\Big)^{\gamma}(r-\mu),$$

$$Q_{F,iso}(Q_L) = \frac{\beta\delta_0(1-\gamma)+Q_L\delta_1+\sqrt{A_{iso}}}{2\delta_1(\beta\gamma-1)},$$

respectively, where

$$A_{iso} = (\beta\delta_0(1-\gamma)+Q_L\delta_1)^2 - 4Q_L\beta(1-\gamma\beta)\delta_0\delta_1.$$

The term $\beta\gamma - 1$ in these functions leads to the following restriction: Only for $\beta\gamma > 1$ it holds that for the iso-elastic demand function the optimal capacity of the follower is positive. Given $X_{F,iso}(Q_L)$ and $Q_{F,iso}(Q_L)$, the value of the leader is given by:

$$V_{L,iso}^{det}(X,Q_L) = \Big(\frac{X_{F,iso}(Q_L)Q_L((Q_L+Q_{F,iso}(Q_L))^{-\gamma}-(Q_L)^{-\gamma})}{r-\mu}\Big)\Big(\frac{X}{X_{F,iso}(Q_L)}\Big)^{\beta} + \frac{XQ_L^{1-\gamma}}{r-\mu} - \delta_1 Q_L - \delta_0.$$

Take the first order partial derivative of $V_{L,iso}^{det}(X,Q_L)$ with respect to Q_L, and set equal to zero. Solving this equation for Q_L gives $Q_L(X)$. We value match the option value of the follower to the value of the leader. The option value of the follower is given by:

$$V_{F,iso}^{option}(X,Q_L) = \Big(\frac{X_{F,iso}(Q_L(X))Q_L(X)(Q_L(X)+Q_{F,iso}(Q_L(X)))^{-\gamma}}{r-\mu} - \delta_1 Q_{F,iso}(Q_L(X)) - \delta_0\Big)\Big(\frac{X}{X_{F,iso}(Q_L(X))}\Big)^{\beta}.$$

The preemption trigger can be obtained by solving the following equation for X_P:

$$V_{L,iso}^{det}(X_P, Q_L(X_P)) = V_{F,iso}^{option}(X_P, Q_L(X_P)).$$

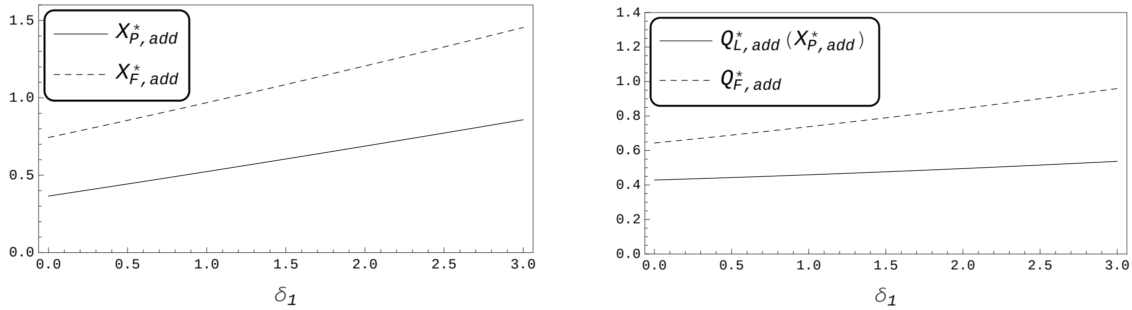
Substitution of X_P^* into $Q_L(X)$ gives $Q_{L,iso}^*(X_P^*)$. Similar, substitution of $Q_{L,iso}^*(X_P^*)$ into $X_{F,iso}(Q_L)$ and $Q_{F,iso}(Q_L)$ give the optimal trigger and capacity of the follower. □

Additional Figures 3.C

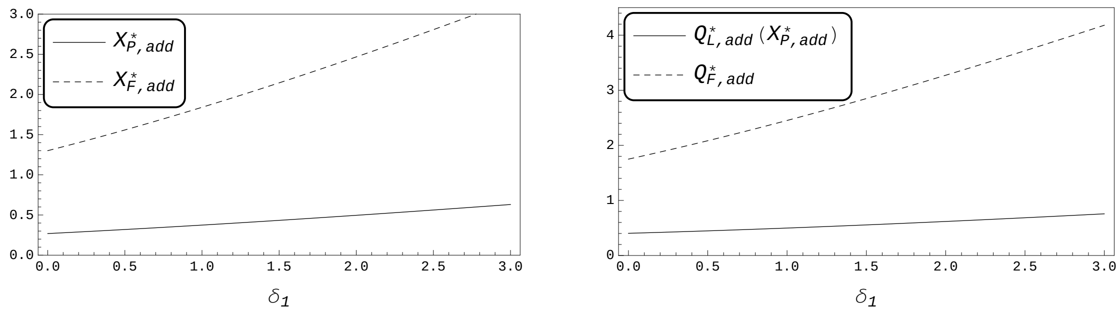
In order to show the robustness of the results from Section 3.4.1, we performed extensive numerical experiments with different parameter values. A sub-sample of this numerical analysis is presented in the following. Regarding additive demand, we show the effect of δ_1 and δ_0 on the optimal timing and capacity choice of a firm for different parameter values (r, μ, σ, η). Similar graphs are given for the multiplicative and iso-elastic demand. Unless stated differently, take parameter values r =0.1, μ =0.02, σ =0.1, η =0.5, γ =0.5, δ_0=0.5, and δ_1 =1.

Additive demand, figures that illustrate changes in δ_1.

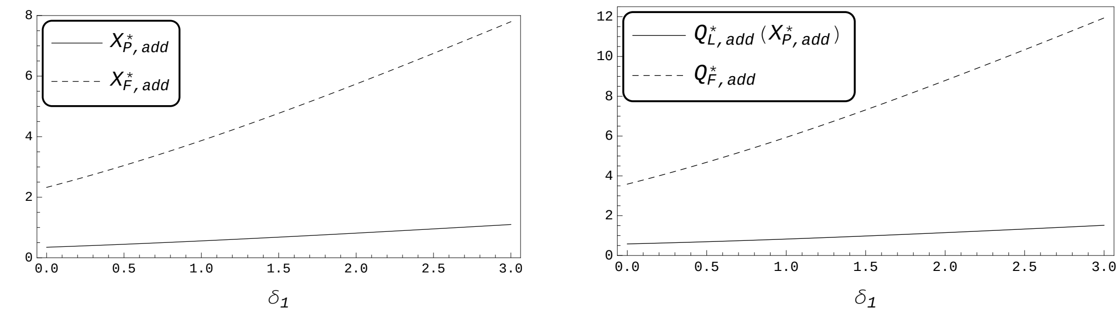
$r = 0.15$



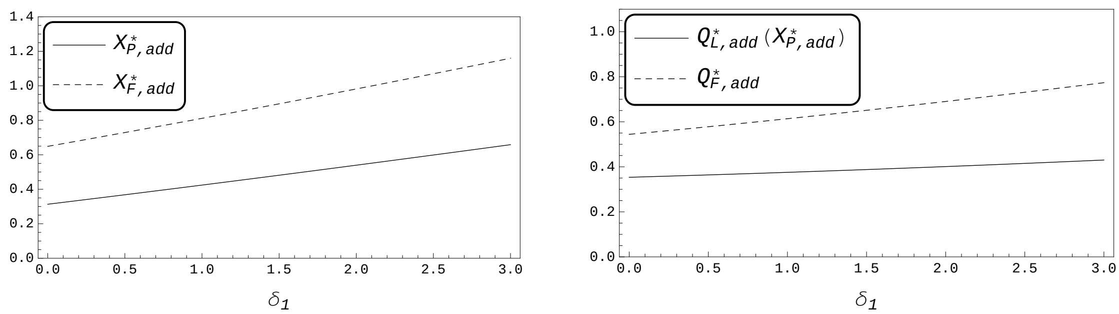
$r = 0.06$



$\mu = 0.04$

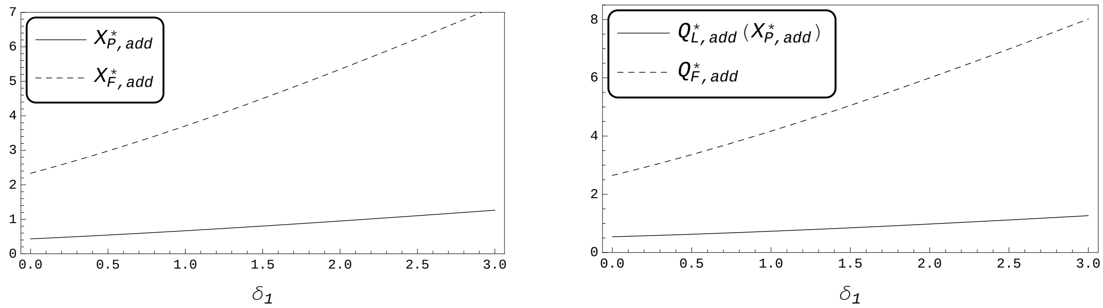


$\mu = 0.01$

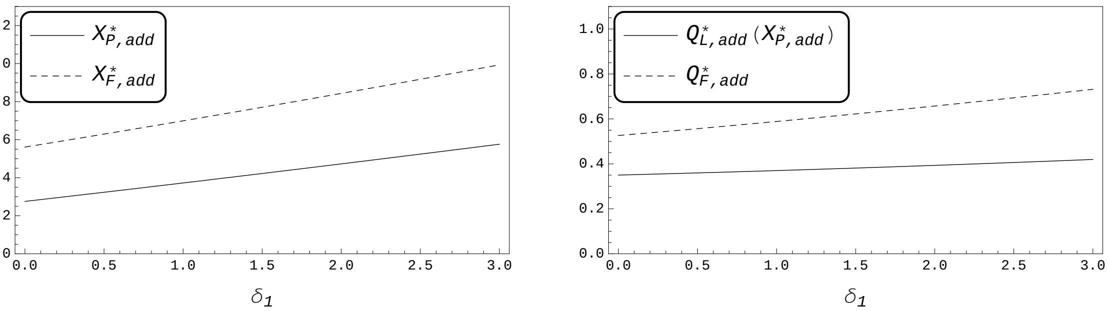


Additive demand, figures that illustrate changes in δ_1.

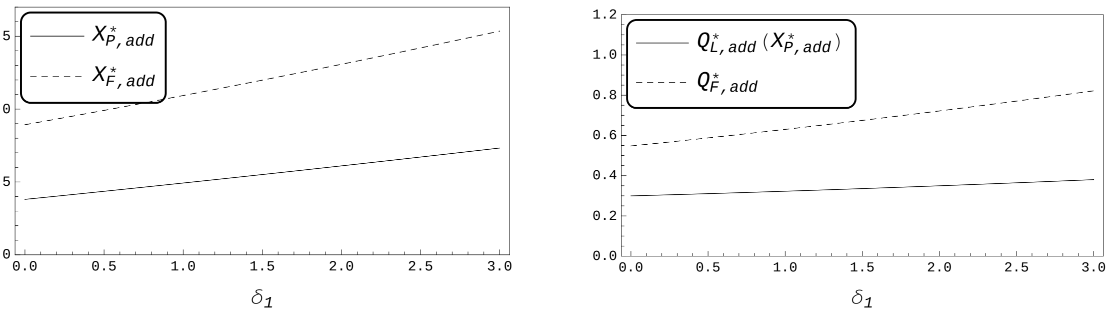
$\sigma = 0.2$



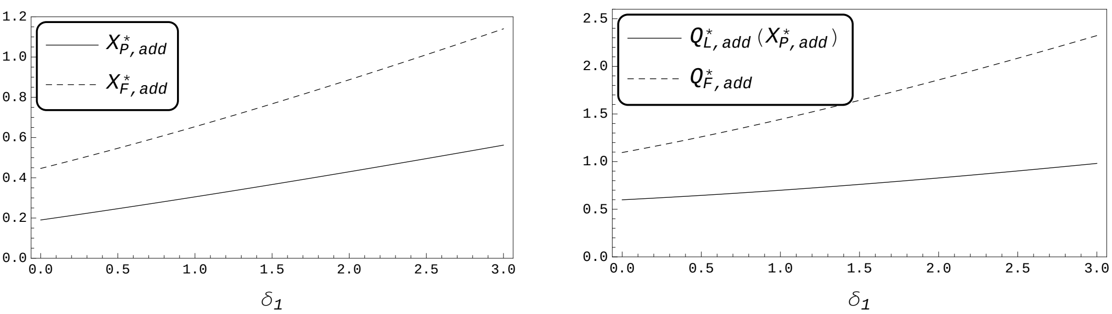
$\sigma = 0.05$



$\eta = 0.8$

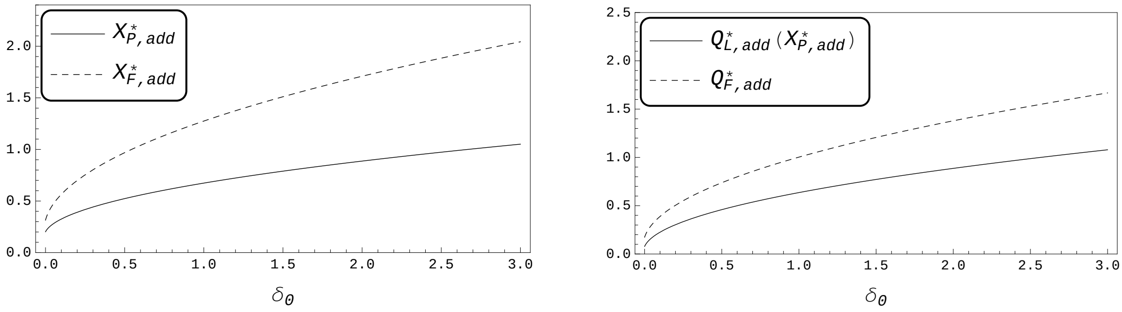


$\eta = 0.2$

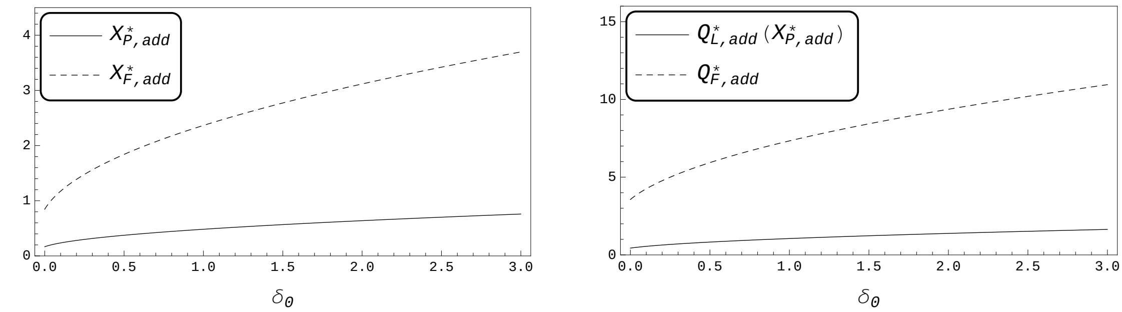


Additive demand, figures that illustrate changes in δ_0.

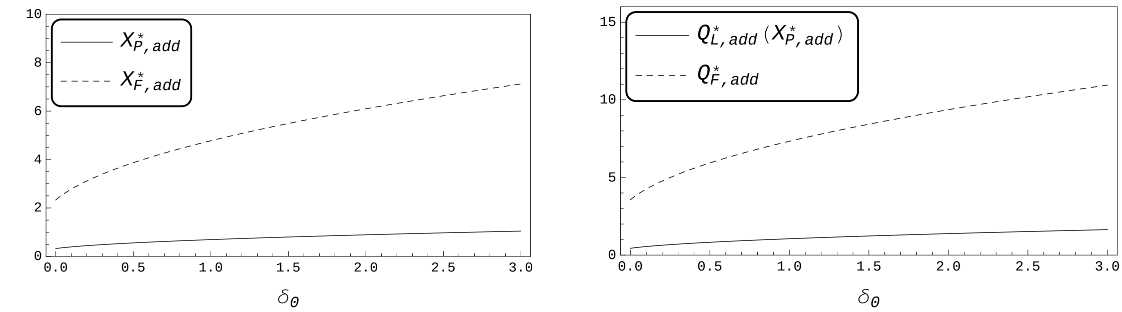
$r = 0.15$



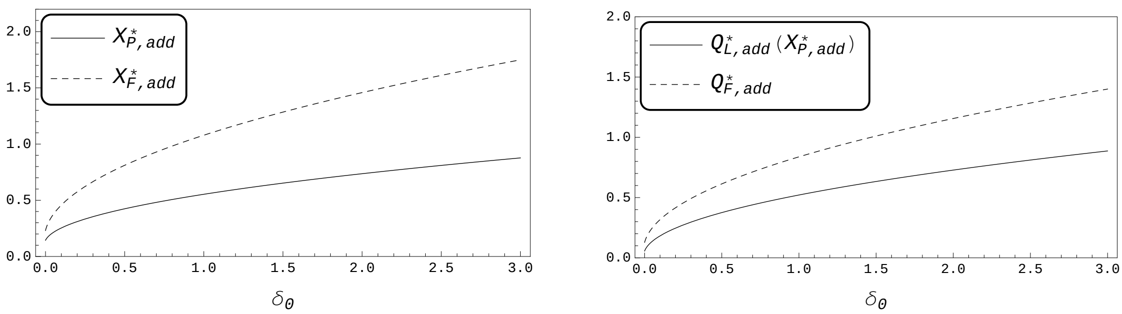
$r = 0.06$



$\mu = 0.04$



$\mu = 0.01$



Additive demand, figures that illustrate changes in δ_0.

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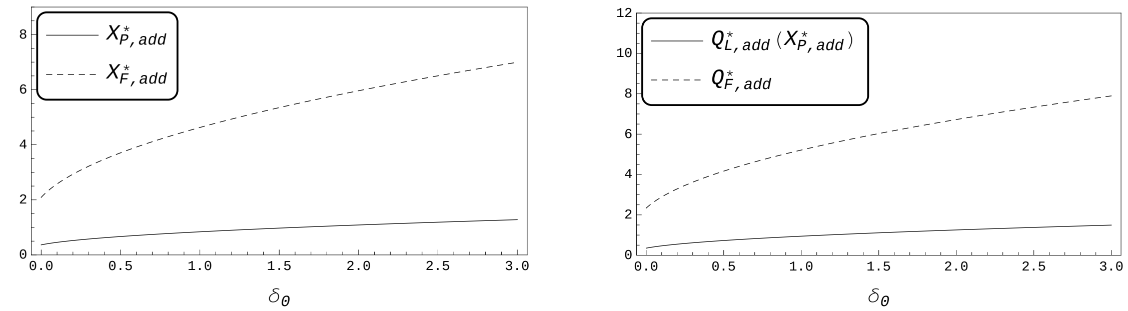

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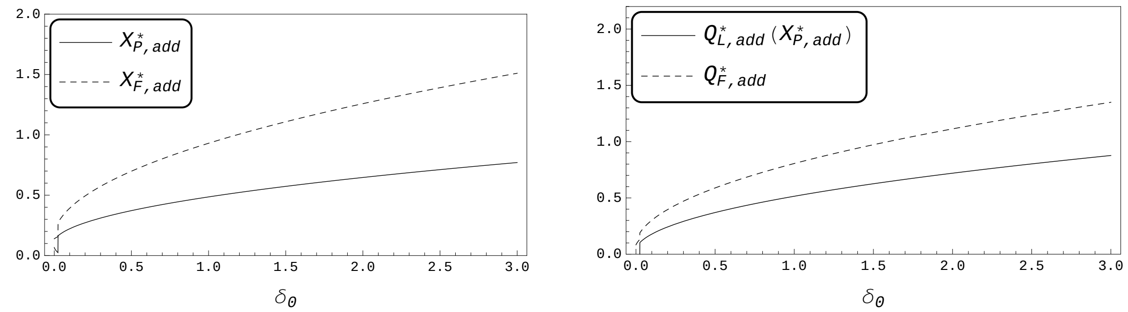

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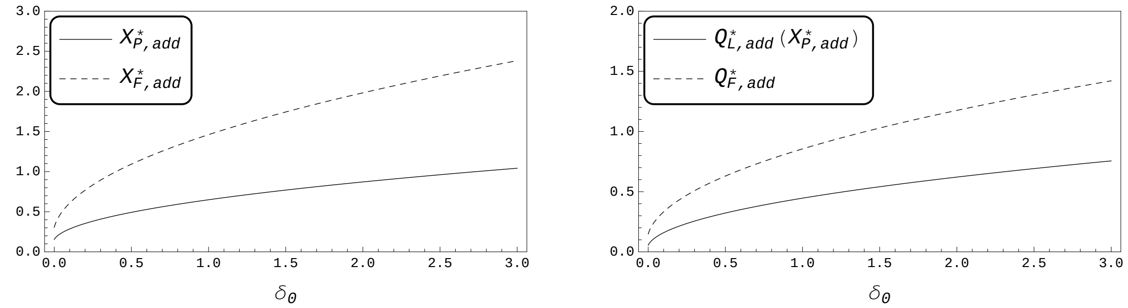

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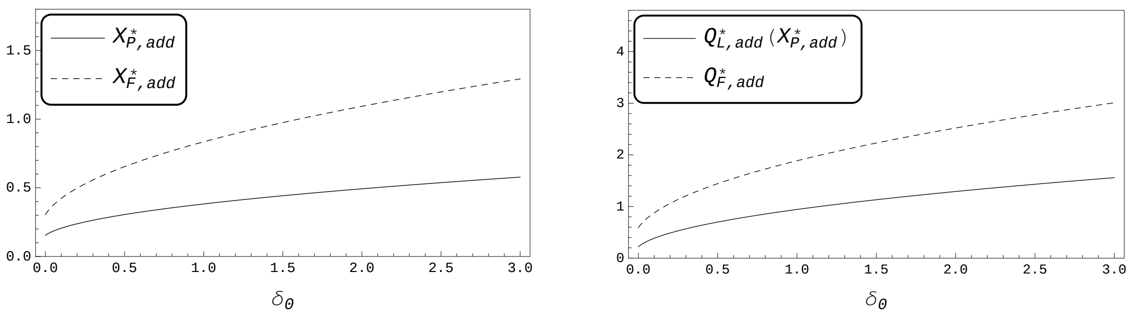

Multiplicative demand, figures that illustrate changes in δ_1.

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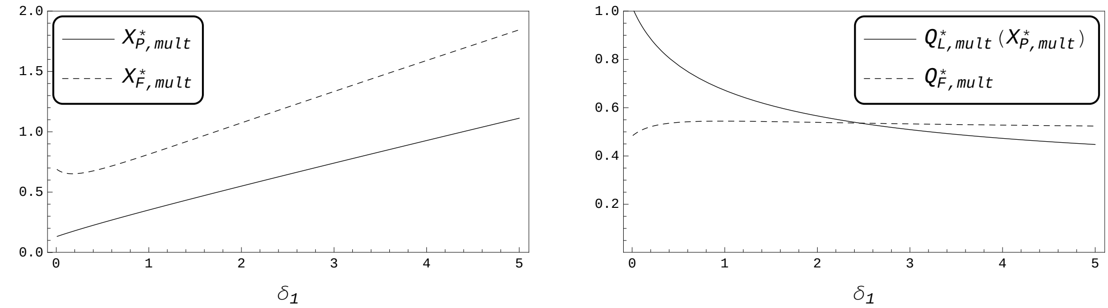

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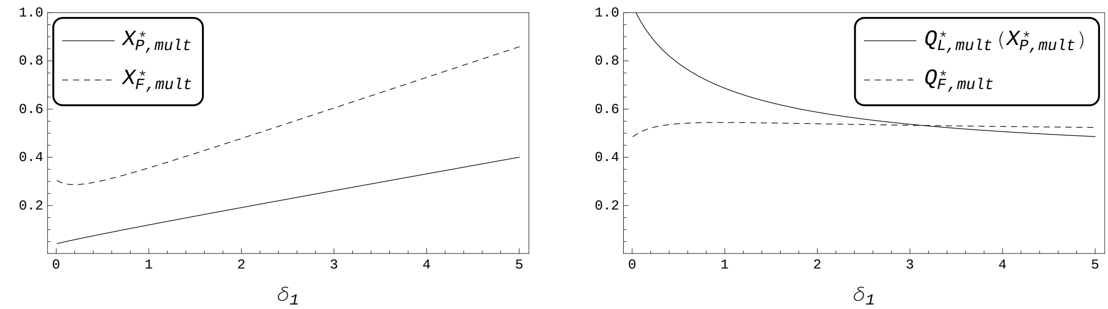

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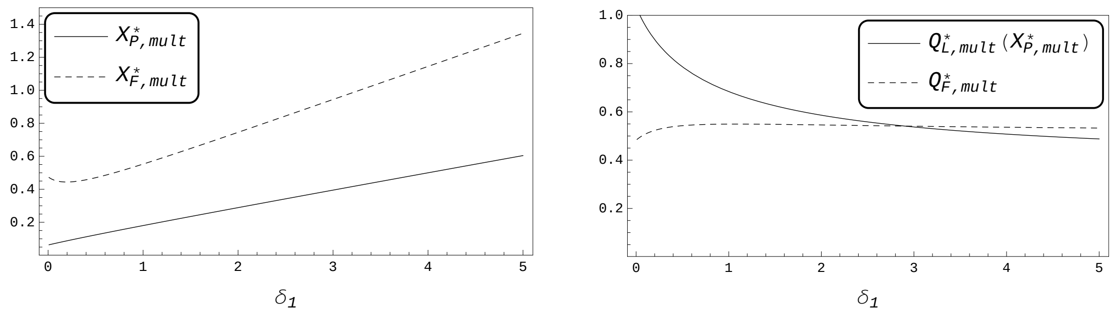

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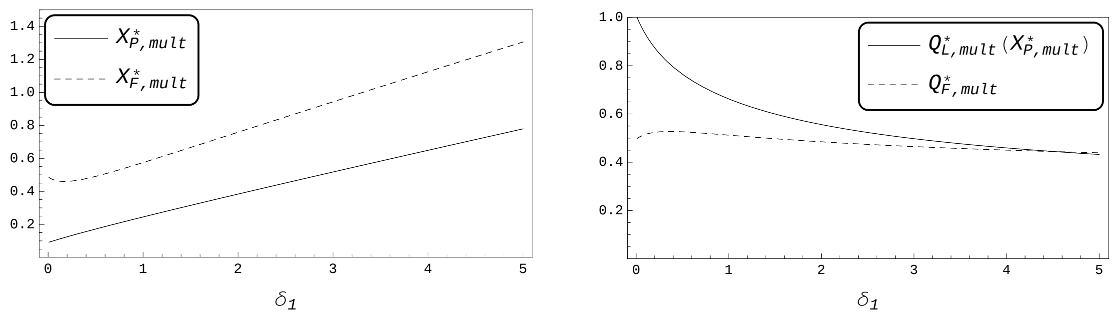
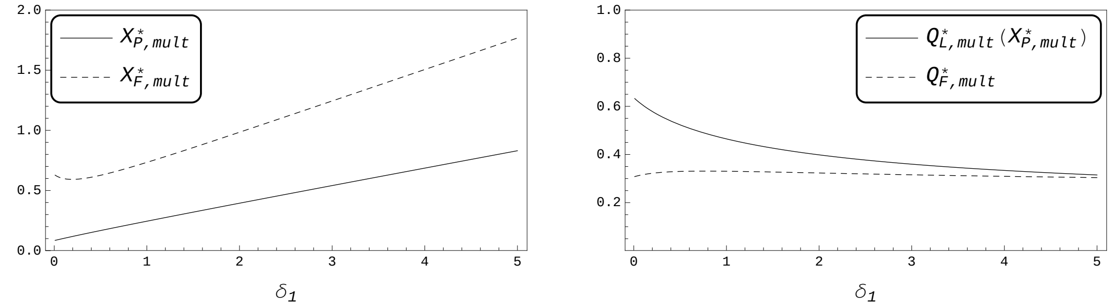


Multiplicative demand, figures that illustrate changes in δ_1.

$\sigma = 0.2$



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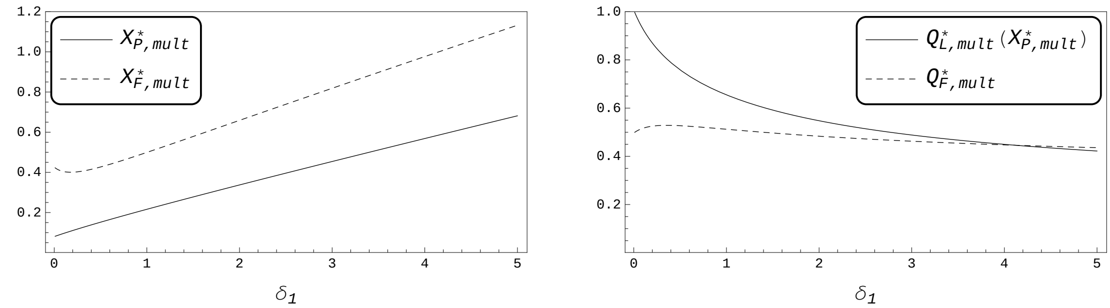

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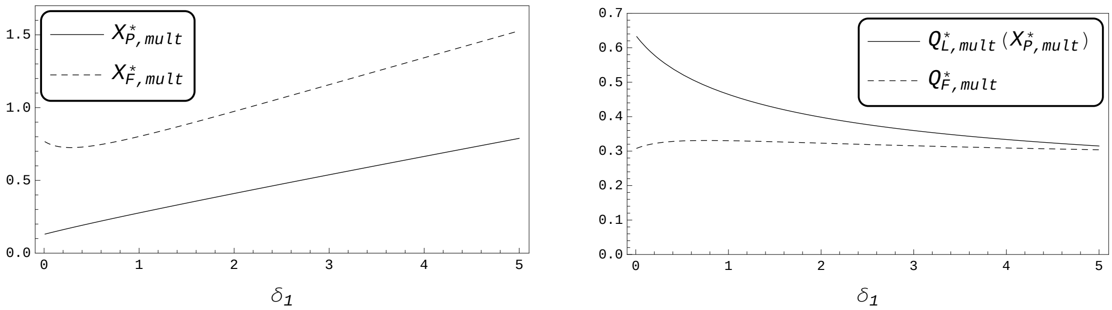

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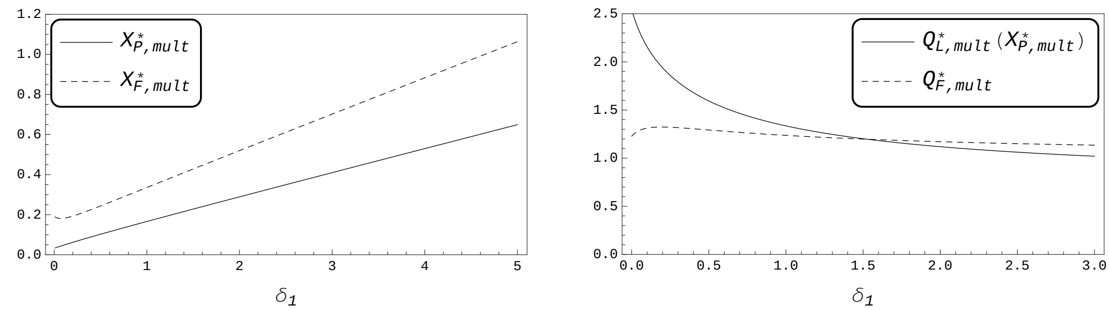

Multiplicative demand, figures that illustrate changes in δ_0.

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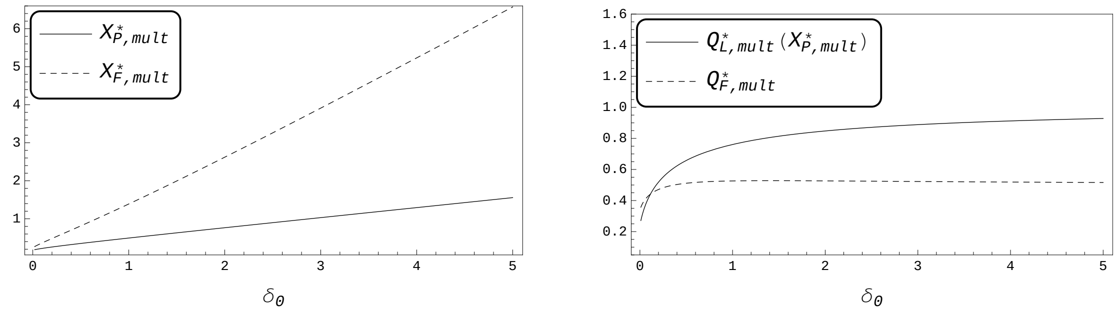

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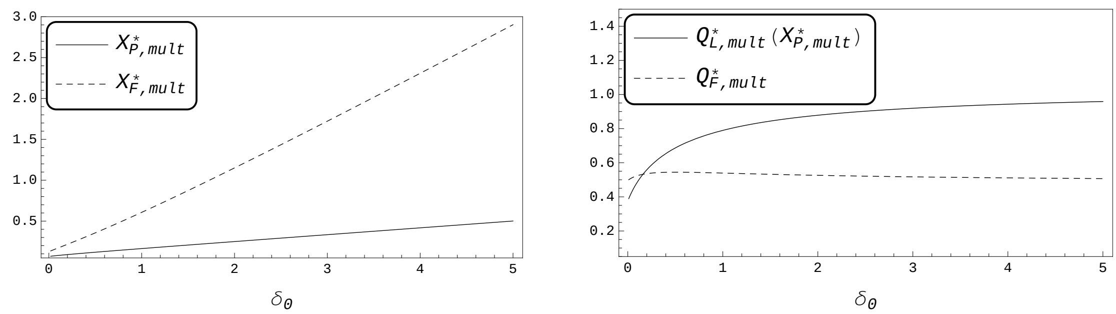

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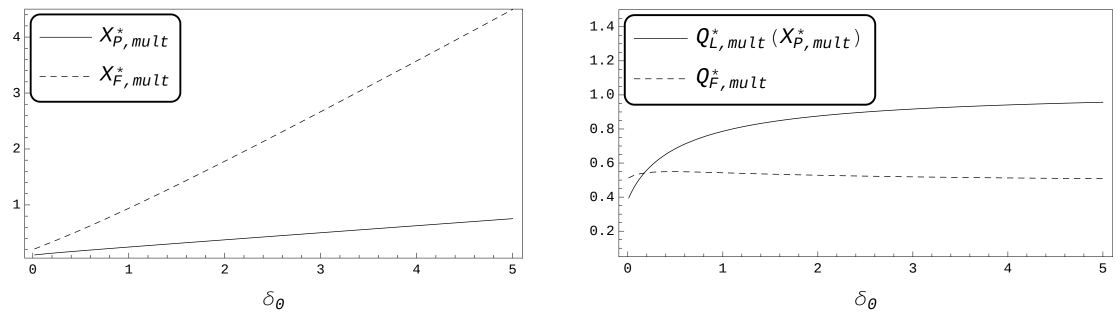

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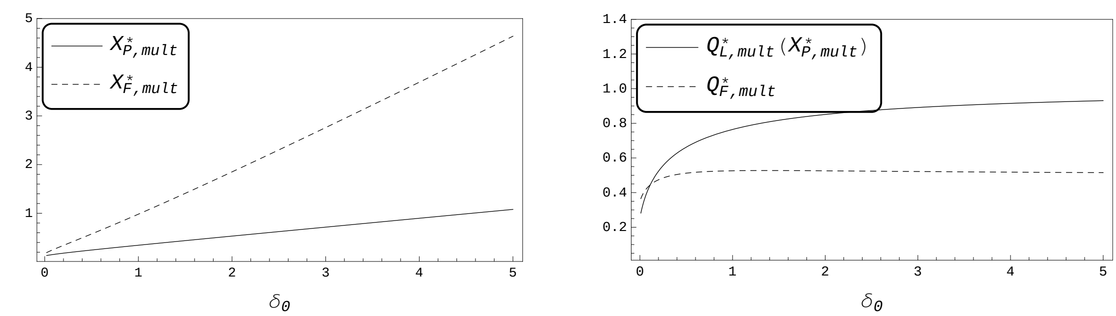
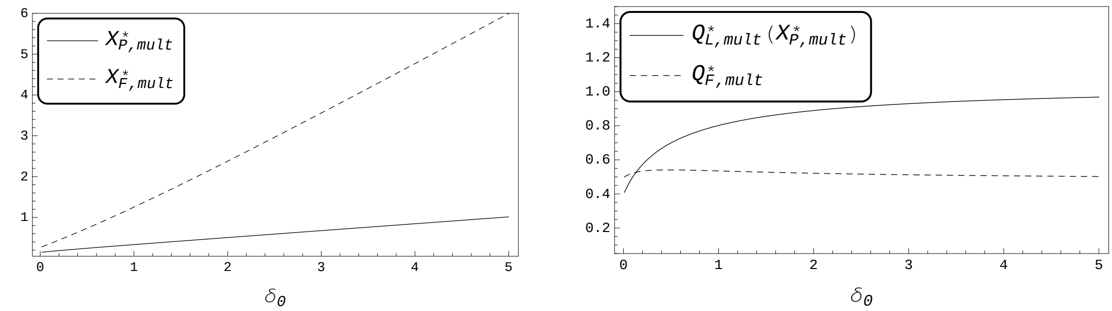


Multiplicative demand, figures that illustrate changes in δ_0.

$\sigma = 0.2$



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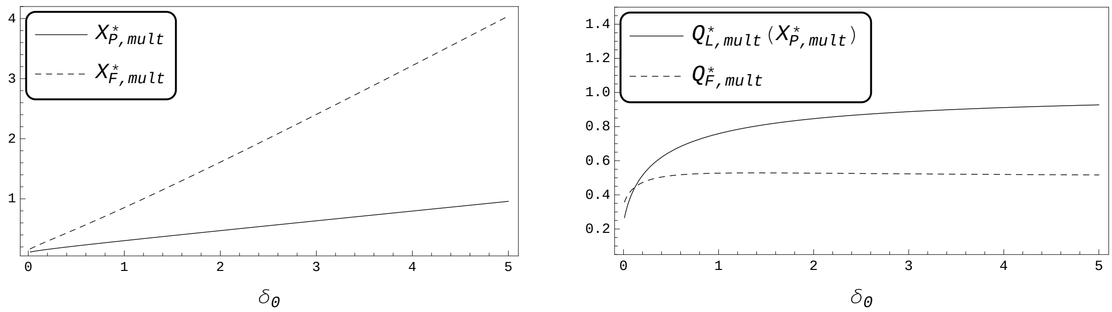

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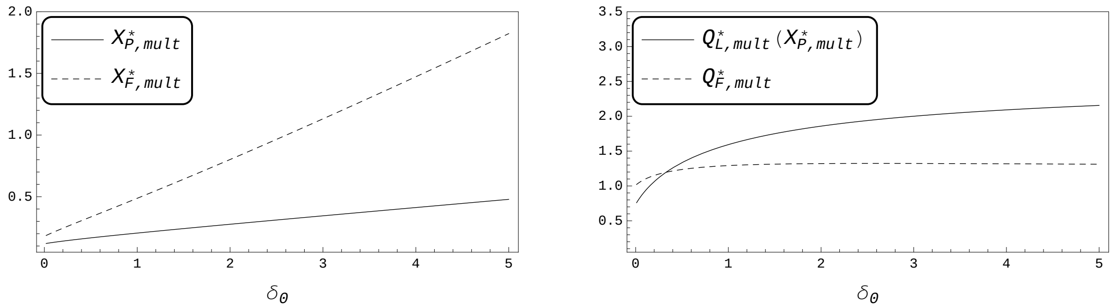

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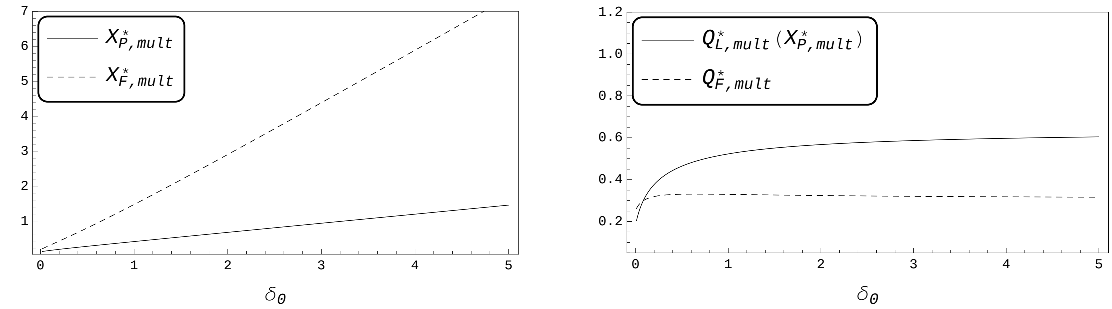
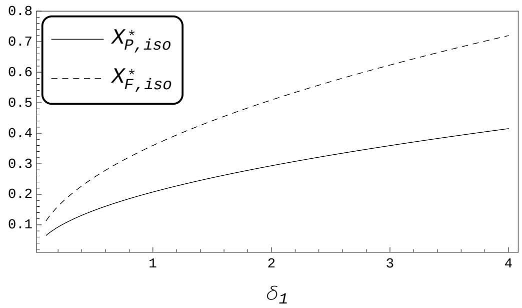
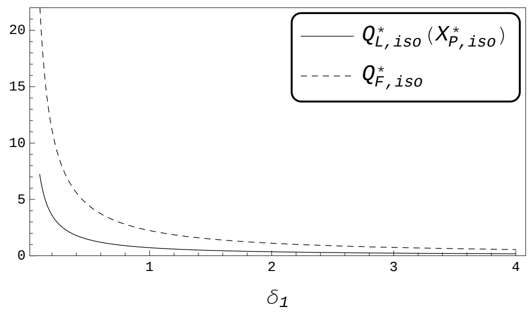


Iso-elastic demand, figures that illustrate changes in δ_1.

$r = 0.15$





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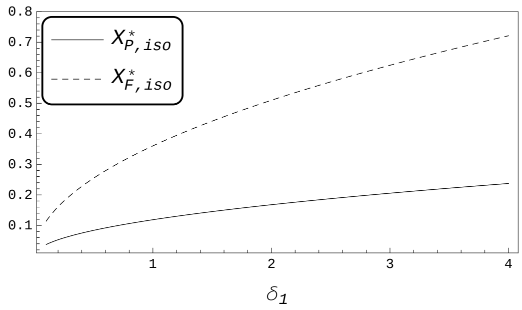
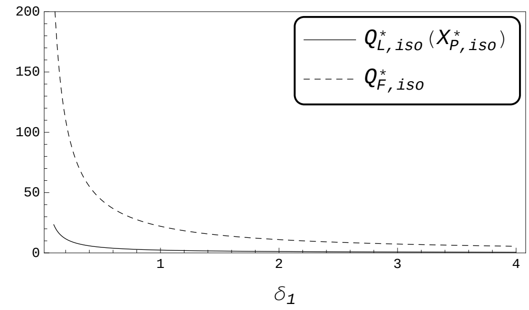
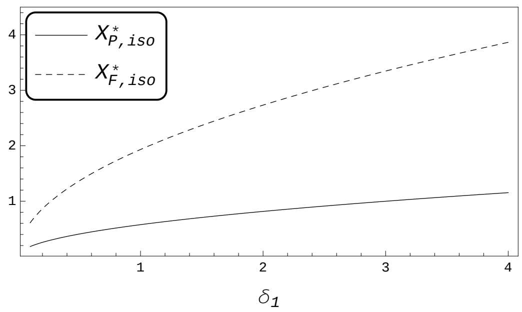
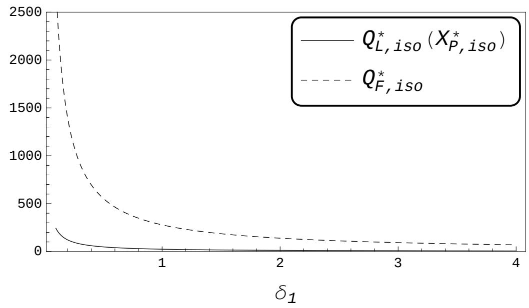


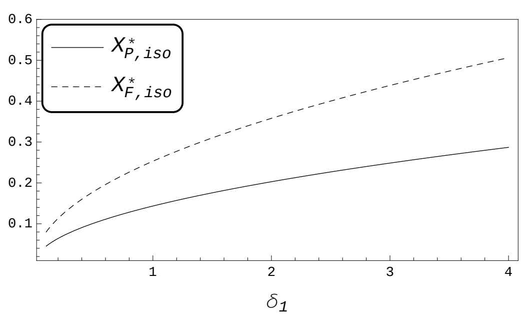


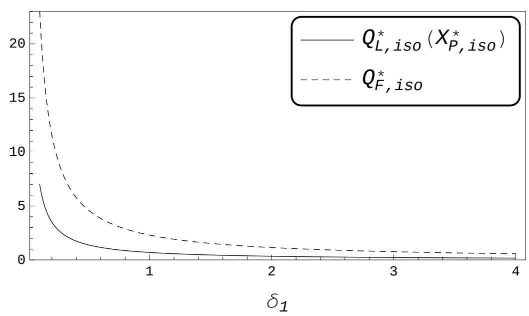
$\mu = 0.04$





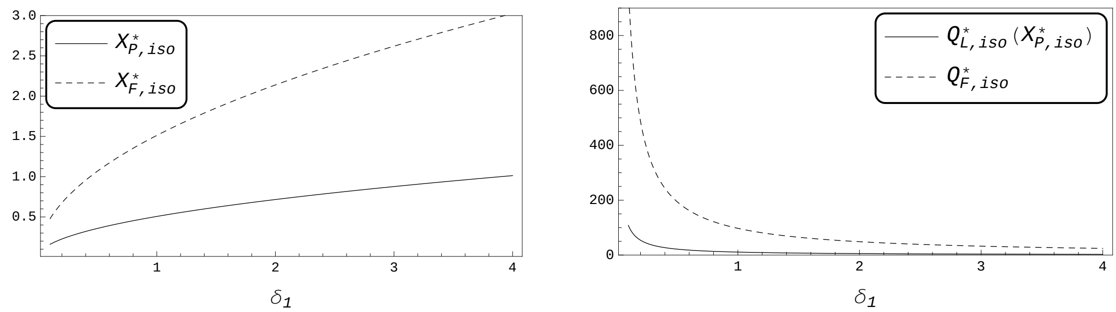
$\mu = 0.01$





Iso-elastic demand, figures that illustrate changes in δ_1.

$\sigma = 0.2$



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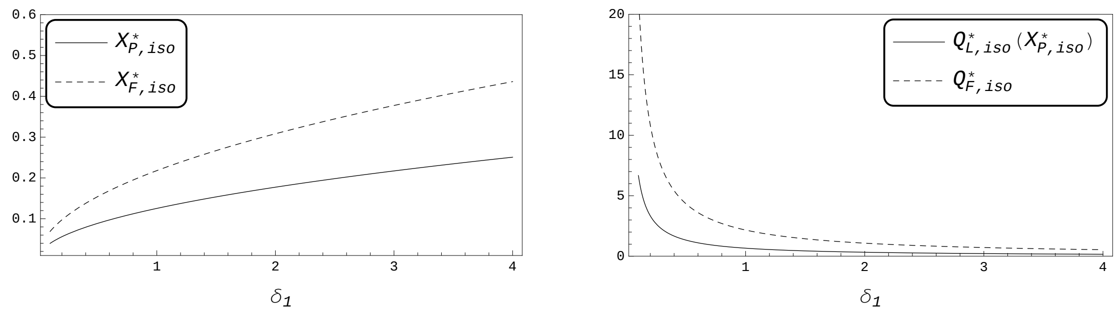

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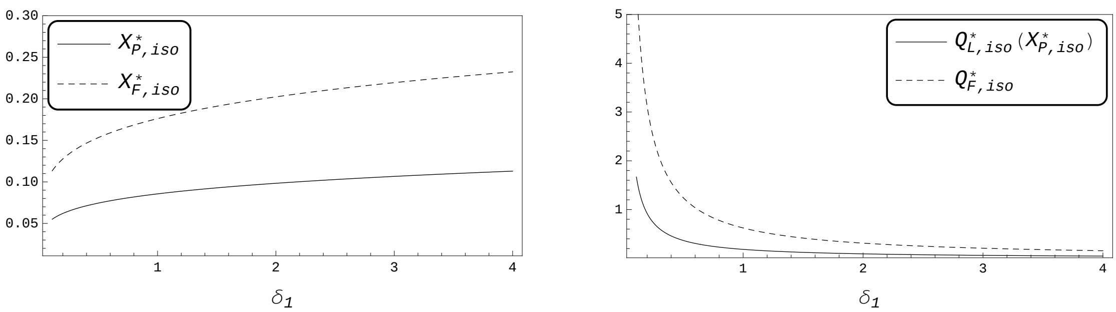

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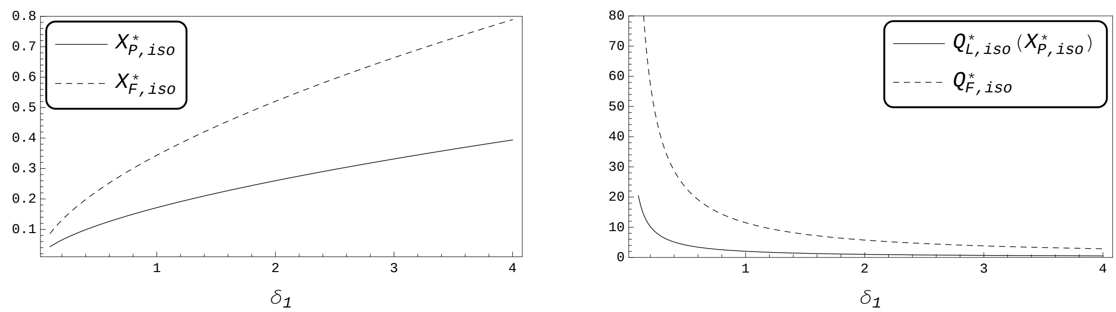
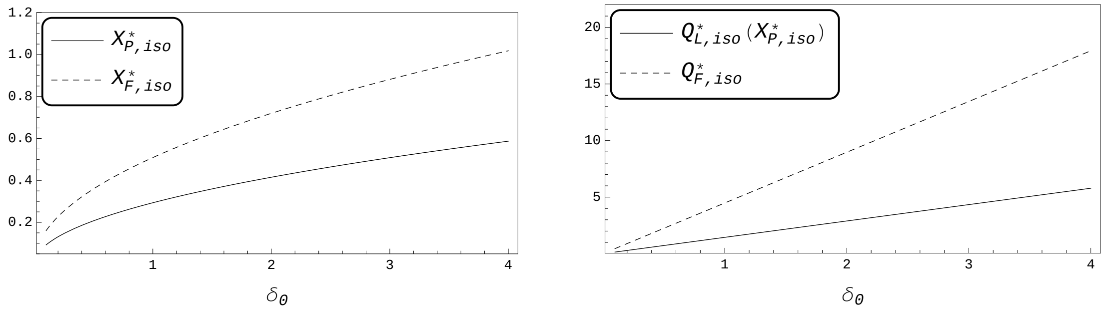
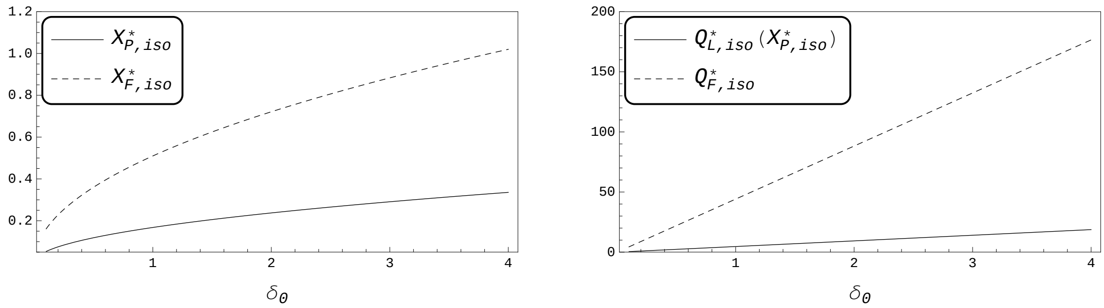


Iso-elastic demand, figures that illustrate changes in δ_0.

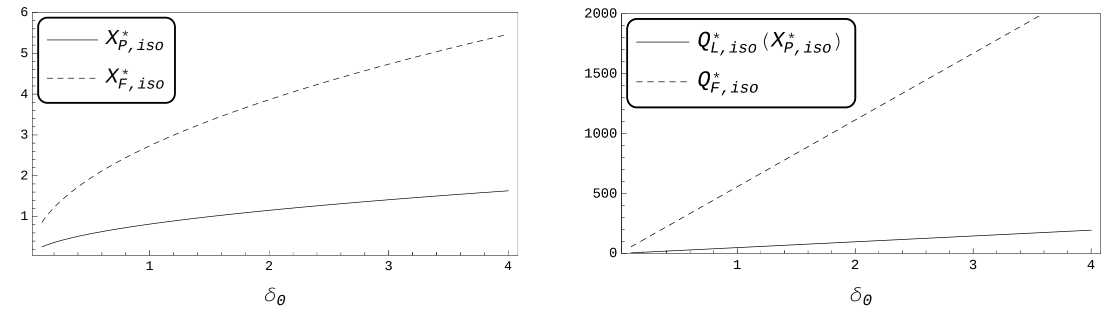
$r = 0.15$



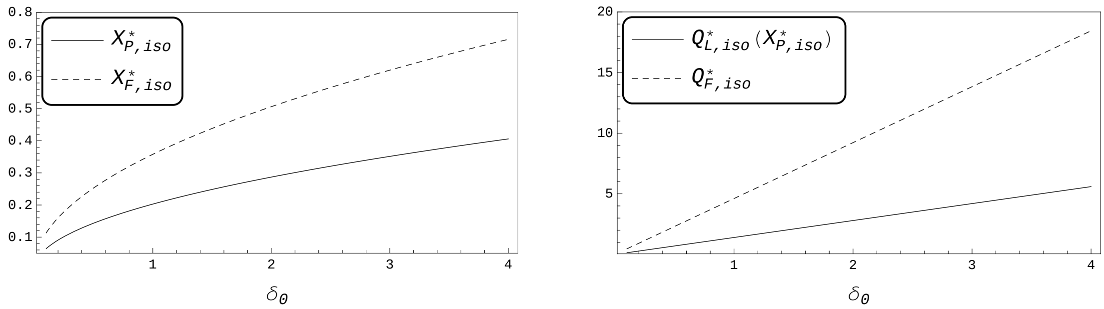
$r = 0.06$



$\mu = 0.04$

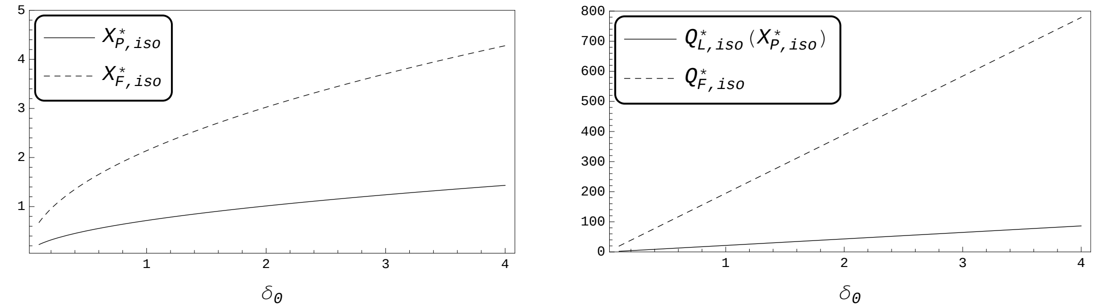


$\mu = 0.01$



Iso-elastic demand, figures that illustrate changes in δ_0.

$\sigma = 0.2$



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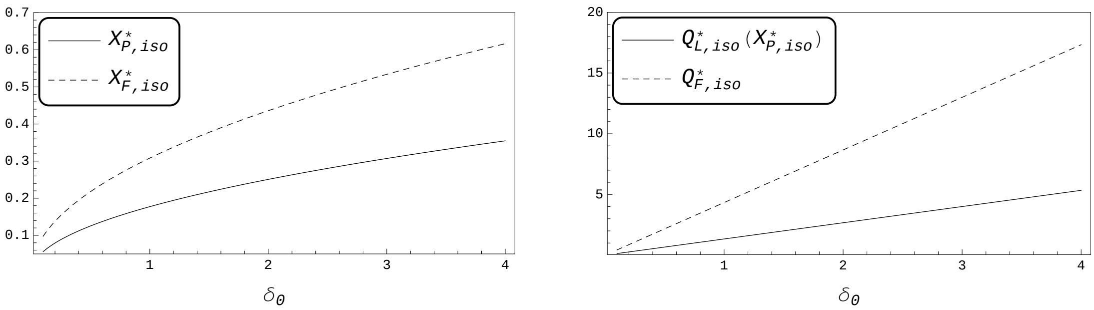

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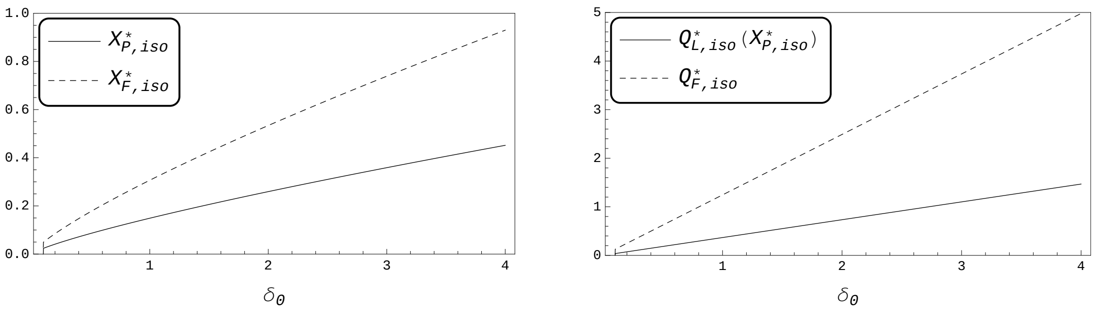

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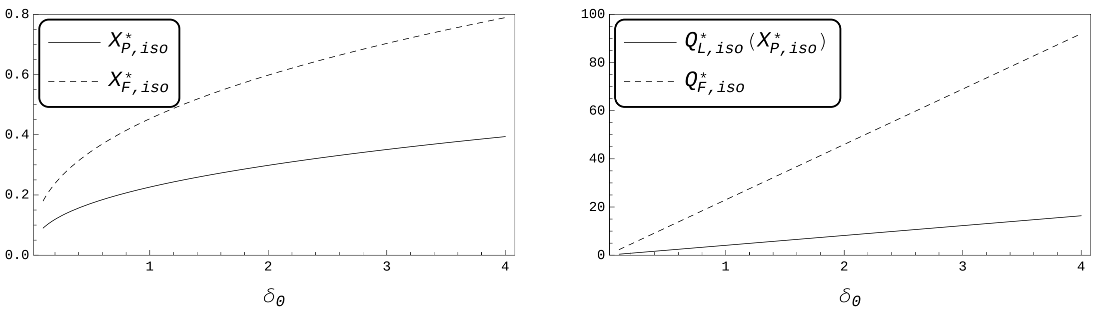

4 Optimal Capacity Investment in Oligopoly: A Real Options Approach

This chapter is based on Boonman et al. (2014b).

Abstract

The purpose of this chapter is to study the effect of demand structures and capacity optimization in a market with three potential entrants. Our chapter makes three specific contributions. First, we show that the accordion effect, derived in Bouis et al. (2009), is sensitive with respect to the demand structure and the presence of capacity choice. The accordion effect can be described as follows. When three firms decide about the optimal moment to enter the market, an increase in the market uncertainty leads to an increase in the third firm's threshold, a decrease in the second firm's threshold and an increase in the first firm's threshold. We find that the accordion effect never occurs under capacity optimization. Instead we have a size effect, where higher uncertainty implies a larger capacity choice which results in a delay of investment. Second, we are able to show analytically that increasing substitutability has a decreasing effect on the capacity choice, while it does not affect the timing of investment. Last, under the assumption of additive demand it can be shown that higher unit capacity costs leads to a later investment in a larger capacity, where under the assumption of multiplicative demand this leads to a later investment, but the optimal capacity choice of the firm remains unaffected.

4.1 Introduction

For many decades already, economists have been interested in the relationship between uncertainty and investment. The traditional approach to value investment projects (the net present value rule) has many flaws. It neglects future uncertainty, deals poorly with the irreversibility of an investment, and assumes a now or never decision. Under the real options approach, the possibility of delay becomes very important in case of an irreversible investment decision, because there exists a high value of waiting. Real options theory argues that a higher level of uncertainty increases the value of a firm's option to invest, it gives a greater incentive to keep this option open, which delays the investment decision. The real options approach, has been pioneered by McDonald and Siegel (1986a), and explained in e.g. Dixit and Pindyck (1994) and Trigeorgis (1996).

While early literature placed its focus on monopoly situations, the lack of strategic interactions in monopoly models makes them unable to take into account the observed competition in industries. A thorough review of strategic investment under uncertainty can be found in Chevalier-Roigant et al. (2011). The focus of this chapter lies on an endogenous oligopoly model with three firms. Fundamental work of Fudenberg and Tirole (1985) explains the principle of rent equalization (resulting from the preemption mechanism) for a duopoly in a deterministic setting. A whole stream of literature built further upon this. Thijssen et al. (2012) extend the duopoly model of Fudenberg and Tirole (1985) by including uncertainty. Other studies that consider the preemption mechanism in a duopoly setting are Huisman and Kort (2004), Nishihara and Fukushima (2008), Weeds (2002), Mason and Weeds (2010), Hoppe (2000), and Pawlina and Kort (2006).

Where this literature analyzes duopoly models, Bouis et al. (2009) explain the importance to combine the real options literature with a framework where the number of firms is larger than two. They state that "in the western economies the extensive process of regulation, combined with a waive of mergers and acquisitions, has resulted in an oligopolistic structure of a large number of sectors". A good example can be found in the energy sector, which experienced a transition from a monopoly supply to a slowly-increasing degree of competition, as a result of successive EU and national legislation designed to liberalise gas and electricity markets.[1]

Our work is closely related to Bouis et al. (2009). They analyze lumpy investment decisions of three symmetric firms that face a simplified version of multiplicative demand. The preemption mechanism determines optimal timing of investment, with cost of investment I. They find that when three firms decide about the optimal time to enter the market, an increase in market uncertainty leads to an increase in the third firm's threshold, a decrease in the second firm's threshold and an increase in the first firm's threshold. Hence, the wedge between the first and the second threshold decreases, and the wedge between the second and the third threshold increases. This is defined as the accordion effect. For a highly uncertain market environment, simultaneous investment between the first two entrants will take place. We elaborate on the results of Bouis et al. (2009) by extending it to a model that includes capacity optimization and different types of demand functions. Below we briefly address these two issues.

Besides timing of investment, also capacity optimization is an important part of the investment decision. Van Mieghem (2003) surveys the strategic capacity management literature. Wu (2007) analyzes a duopoly model with multiplicative demand. That paper obtains that in most cases the first mover equilibrium strategy is to enter with a smaller capacity than the leader. Huisman and Kort (2014) also consider multiplicative demand and they show that the level of uncertainty determines which firm has the biggest capacity in the market. A highly uncertain market environment results in a market where the leader invests in the largest capacity. A low market uncertainty, on the contrary, results in a market where the follower obtains the biggest part of the market share. An important difference between these two models is that in Wu (2007) uncertainty only lies at the switching point of market growth to market decline, where in Huisman and Kort (2014) market size is subject to uncertainty at any point.

Considering the effect of different demand structures on the firm's investment decisions, Anupindi and Jiang (2008) and Boonman and Hagspiel (2014) are two other papers that shed light on investment decisions from the per-

[1] http://www.cliffordchance.com/content/dam/cliffordchance/PDFs/Mergers_in_the_energy_sector_e_competitions.pdf

spective of different demand structures. Anupindi and Jiang (2008) categorize inverse demand functions into two streams for different demand shocks, namely multiplicative demand and additive demand. Boonman and Hagspiel (2014) include another version of multiplicative demand in their analysis, known as iso-elastic demand. Even though Boonman and Hagspiel (2014) consider uncertainty at any point in time (just like in Huisman and Kort (2014)), they confirm the result found by Wu (2007) under additive or iso-elastic demand, contrary to what Huisman and Kort (2014) obtain with multiplicative demand.

Other extensions of Bouis et al. (2009) are Argenziano and Schmidt-Dengler (2014) and Zhang et al. (2013). Argenziano and Schmidt-Dengler (2014) consider a model where a finite number of firms optimize timing of an irreversible investment. They find that for a sufficiently intense preemption race among late investors, the preemption incentive for earlier investors disappears and two or more investments occur at the same time. Like Bouis et al. (2009) they use a simple payoff function, but do not include uncertainty in the model. Zhang et al. (2013) present a model with an additive demand structure, where three firms optimize capacity. Production flexibility is incorporated by the assumption of a price cap. We extend Zhang et al. (2013) by incorporating endogenous firm roles.

This chapter considers a new market model, where three firms optimize the level of capacity and the timing of investment. Two types of demand functions are considered: multiplicative and additive demand. The aim is to show whether the accordion effect, found by Bouis et al. (2009), will hold under the assumption of different demand structures and capacity optimization.

We find, under capacity optimization, that the accordion effect as well as an equilibrium where firms invest simultaneously, do not exist. Firms are asymmetric ex post investment due to the choice of different optimal capacities. Pawlina and Kort (2006), however, find a simultaneous equilibrium under asymmetric firm roles. But they acknowledge that this result only occurs due to the existing market assumption. Since we allow firms to optimize capacity, there is another effect that plays a role, namely the size effect. The size effect, which has been recognized by other contributions in the literature (Huisman and Kort (2014), Dangl (1999), Bar-Ilan and Strange (1999) and Manne (1961)), states that higher levels of uncertainty result in a delayed investment and a relatively larger investment size. We find that this size effect is stronger than the accordion effect, i.e. the investment gap between all investors increases in a more uncertain market environment. As a last contribution, we are able to present an analytical solution for the firm's investment decision. Hence, we are able to prove analytically that an increasing substitutability parameter does not affect the timing of a firm's investment and decreases its optimal capacity choice. For additive demand we can show that increasing variable investment costs result in larger investments, which, however, take place at later points in time. Also under multiplicative demand, increasing variable investment costs delay the moment of investment but the optimal capacity choice of the firms are not affected.

This chapter is organized as follows. The general model is presented in Section 4.2. Section 4.3 describes the value functions and investment decisions of the three firms, where in the first subsection multiplicative demand is considered and in the second subsection additive demand. In Section 4.4 the results are discussed, and Section 4.5 concludes.

4.2 The model

This chapter considers an oligopoly market consisting of three ex ante identical firms that produce a homogeneous good. Upon investment, the firms have the opportunity to optimize the timing of investment and determine the optimal level of capacity. Firms have an incentive to be the first entrant, due to the resulting temporary period of monopoly profits. The remaining firms preempt each other in order to enjoy a period of duopoly profits corresponding to the second entrant's investment time. For the last firm's position, there is no entry threat of a future competitor, and this firm invests at its optimal investment trigger, not being influenced by strategic aspects. This situation corresponds to endogenous firm roles, i.e. firms do not know beforehand what will be their position in this market (first, second or third investor), which is a realistic reproduction of reality.

Unit investment costs are denoted by c. Investor i, $i \in \{1,2,3\}$, which invests in capacity $Q_i > 0$, will therefore have to pay total investment costs cQ_i. Once investment is performed, this firm receives a profit flow equal to

$$\pi_{t,i} = P_t Q_{t,i},$$

where P_t is the price at time t. We will consider two types of demand structures, namely multiplicative demand and additive demand, which are given by

$$P_t = X_t(1 - \eta Q_t) \tag{4.2.1}$$

and

$$P_t = X_t - \eta Q_t, \tag{4.2.2}$$

respectively, where Q_t is the total supply of the product in the market. If there are two firms in the market for instance, the total supply is equal to $Q_{t,1} + Q_{t,2}$. X_t is assumed to follow a geometric Brownian motion process

$$dX_t = \mu X_t dt + \sigma X_t d\omega_t,$$
$$X_0 = x,$$

with drift parameter μ, volatility parameter σ and $d\omega_t$ representing the increment of a standard Wiener process. The inverse demand functions in equations (4.2.1) and (4.2.2) are also known as the net-price functions. The net price functions are equal to the gross price functions, p_t, subtracted by the variable costs, i.e. $P_t = p_t - \delta$. δ denotes the variable cost per unit of production. The additive demand structure could lead to a negative price, which would simply imply that the firm's production costs exceed its revenue. Furthermore, the firms are assumed to be risk neutral and maximize the expected discounted cash flow stream. Firms have a constant discount rate r, such that $r > \mu$.

We assume that a firm produces up to capacity. This concept, known in the literature as the "market clearance assumption", is widely used in the literature (Chod and Rudi (2005), Deneckere et al. (1997), Anand and Girotra (2007), and Goyal and Netessine (2007)).

4.3 Model Analysis

Bouis et al. (2009) show the accordion effect considering a specific type of multiplicative demand. Namely, the profit flow of entrant k is given by $\pi_k = Y(t)D_k$, where $Y(t)$ follows a geometric Brownian motion and the effect of competition on the profit is reflected in D_k. This chapter considers two different types of inverse demand structures, multiplicative and additive demand structure as in (4.2.1) and (4.2.2), respectively. In this section we analyze the effect of different demand structures and capacity optimization on the occurrence of the accordion effect. We consider multiplicative demand in Subsection 4.3.1 and additive demand in Subsection 4.3.2.

Multiplicative Demand 4.3.1

Sequential Investment
In a sequential game, the first investor obtains a period of monopoly profits, and after the second investor entered, a period of duopoly profits. After the entry of the third firm, all firms obtain an oligopoly profit, where the market is shared with three firms. The value of investor i, $i \in \{1,2,3\}$, is denoted by $V_i(X)$. This firm invests in capacity size Q_i at the optimal investment moment X_i. Due to the market clearance assumption, the firms produce different quantities of the product if their corresponding capacities differ. Obviously, this also leads to different profit flows of firms in the same market position (duopoly or oligopoly). Here our model differs from the model of Bouis et al. (2009), where firms in the same market situation obtain the same profit.

According to the standard approach to solve dynamic games, we analyze the problem backwards in time. Assuming that the first two investors have invested already, we first analyze the investment decision of the third investor. Due to the absence of future entrants that could preempt the third firm's market position, there are no strategic aspects involved in this investment decision. The third investor's value function, $V_3(X)$, is given by

$$V_3(X,Q_1,Q_2,Q_3) = \begin{cases} (\frac{X}{X_3})^{\beta_1}\left(\frac{X_3 Q_3(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_3\right) & X < X_3, \\ \frac{XQ_3(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_3 & X \geq X_3, \end{cases} \tag{4.3.1}$$

where $\beta_1 > 1$ is the positive solution of the so-called fundamental quadratic, i.e. $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$ (see e.g. Dixit and Pindyck (1994)). The term in the first row of (4.3.1) represents the 'value of waiting' to become the third investor ($W_3(X,Q_1,Q_2,Q_3)$). The value of waiting is given by the discounted net present value of the revenue after investment, where $(\frac{X}{X_3})^{\beta_1}$ is the stochastic discount factor (Dixit and Pindyck (1994)). The following proposition presents the optimal investment threshold X_3 of the third investor. All proofs of subsequent propositions can be found in the appendix.

Proposition 4.1
Consider multiplicative demand. The third investor invests as soon as X reaches the optimal investment trigger X_3, given by

$$X_3 = \frac{c\beta_1(r-\mu)}{(\beta_1-1)(1-\eta(Q_1+Q_2+Q_3))}, \tag{4.3.2}$$

and invests in the following optimal level of capacity

$$Q_3 = \frac{c(1-\eta(Q_1+Q_2))}{\eta(\beta_1^2-1)}.$$

Next, we analyze the investment decision of the second investor, considering that the first investor has invested already. The value of the second investor, $V_2(X,Q_1,Q_2,Q_3)$, is given by

$$V_2(X,Q_1,Q_2,Q_3) = \begin{cases} (\frac{X}{X_3})^{\beta_1}(-\frac{\eta Q_2 Q_3 X_3}{r-\mu}) + \frac{XQ_2(1-\eta(Q_1+Q_2))}{r-\mu} - cQ_2 & X < X_3, \\ \frac{XQ_2(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_2 & X \geq X_3. \end{cases} \tag{4.3.3}$$

The first row of (4.3.3), describes the scenario where the third entrant has not invested yet, the second entrant therefore earns a duopoly profit with optimal capacity size Q_2. However, this firm knows that at X_3, it will also have to share the market with the third entrant, which decreases its profit by $\frac{\eta Q_2 Q_3 X_3}{r-\mu}$. The value of the second entrant, after the entry of the third investor, is stated in the second row of (4.3.3). Notice that this profit is not the same as the oligopoly profit of the third entrant, since they produce different quantities.

The second entrant will invest at the moment that it is indifferent between becoming the second entrant and waiting to be the third entrant. This occurs at the preemption trigger which is denoted by X_2. When X falls below this trigger, both firms prefer the third entrant's position. However, when a firm waits until after X_2 with investment, the value of immediate investment is higher than the value of waiting, and the other firm will preempt by investing just slightly sooner. This game will continue until $W_3(X_2,Q_1,Q_2,Q_3) = V_2(X_2,Q_1,Q_2,Q_3)$, where $W_3(X,Q_1,Q_2,Q_3)$ is the value of waiting of becoming the third entrant in the market (first line of (4.3.1)), and $V_2(X, Q_1, Q_2, Q_3)$ is the value of investing secondly (first line of (4.3.3)). This implies that the expected payoffs are equal in equilibrium, which is called 'rent equalization' in the literature (Fudenberg and Tirole (1985)). Along with optimizing the timing of investment, the second investor also optimizes its level of capacity. That is, it takes the first order condition of the top row of its value function ($V_2(X_2, Q_1, Q_2, Q_3)$) with respect to its capacity size (Q_2) and finds the root that leads to a maximum value. The following proposition states the equations that need to be solved simultaneously in order to find the optimal investment decision of the second entrant.

Proposition 4.2
Consider multiplicative demand. The preemption trigger X_2 and optimal capacity level Q_2 of the second investor are implicitly defined by

$$\left(\frac{X_2}{X_3}\right)^{\beta_1}\left(\frac{X_3Q_3(1-\eta(Q_1+Q_3))}{r-\mu} - cQ_3\right) \\ - \frac{X_2Q_2(1-\eta(Q_1+Q_2))}{r-\mu} + cQ_2 = 0 \quad (4.3.4)$$

and

$$\left(\frac{X_2}{X_3}\right)^{\beta_1}\left(\frac{-c^2(1-\eta(Q_1+Q_2)+\eta Q_2\beta_1)}{(1-\eta(Q_1+Q_2))(1-\beta_1^2)^2}\right) \\ + \frac{X_2(1-\eta(Q_1+Q_2))}{r-\mu} - c = 0. \quad (4.3.5)$$

Finally we analyze the investment decision of the first entrant. The value function of the first investor is given by:

$$V_1(X, Q_1, Q_2, Q_3) = \\ \begin{cases} (\frac{X}{X_3})^{\beta_1}(-\frac{\eta Q_1Q_3X_3}{r-\mu}) + (\frac{X}{X_2})^{\beta_1}(-\frac{\eta Q_1Q_2X_2}{r-\mu}) \\ \qquad + \frac{XQ_1(1-Q_1\eta)}{r-\mu} - cQ_1 & X < X_2, \\ (\frac{X}{X_2})^{\beta_1}(-\frac{\eta Q_2Q_3X_3}{r-\mu}) + \frac{XQ_1(1-\eta(Q_1+Q_2))}{r-\mu} - cQ_1 & X_2 < X < X_3, \\ \frac{XQ_1(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_1 & X \geq X_3, \end{cases} \quad (4.3.6)$$

where the first row represents the monopoly profit of the first investor including the negative revenue shock at X_2 and X_3 when the second and third investor enter the market, respectively. The second line of (4.3.6) represents the duopoly profit of the first investor, which differs from the duopoly profit of the second investor due to different capacity choices. The oligopoly value of the first investor, after the entry of the final firm is given by the term in the third row of (4.3.6).

Initially, it is not yet clear which firm will become the first, second or third investor in the market. All firms try to be in the position of the first investor, motivated by the period of monopoly profits. The first investor invests when it is indifferent between waiting for the position of the second entrant and investing immediately. This preemption moment is denoted by X_1. The value of waiting for the second firm position is defined by[2]:

$$W_2(X,Q_1,Q_2,Q_3) = \left(\frac{X}{X_2}\right)^{\beta_1}\left(\left(\frac{X_2}{X_3}\right)^{\beta_1}\left(-\frac{\eta Q_2 Q_3 X_3}{r-\mu}\right)+\frac{X_2 Q_2(1-\eta(Q_1+Q_2))}{r-\mu}-cQ_2\right).$$

Preemption trigger X_1 is found by solving $V_1(X_1,Q_1,Q_2,Q_3) = W_2(X_1,Q_1,Q_2,Q_3)$. Additionally, the firm optimizes the capacity size in which it will invest. For this, take the first order condition of the first row of (4.3.6) with respect to Q_1, the root for which this value function obtains a maximum results in the firm's optimal capacity choice. The following proposition states the two equations that lead to the optimal investment decision of the market leader.

Proposition 4.3

Consider multiplicative demand. The preemption trigger X_1 and optimal capacity level Q_1 of the first investor are implicitly defined by

$$\left(\frac{X_1}{X_3}\right)^{\beta_1}\left(\frac{X_3 Q_3 \eta(Q_1-Q_2)}{r-\mu}\right)+\left(\frac{X_1}{X_2}\right)^{\beta_1}\left(\frac{X_2 Q_2(1-\eta Q_2)}{r-\mu}-cQ_2\right) - \frac{X_1 Q_1(1-\eta Q_1)}{r-\mu}+cQ_1 = 0, \quad (4.3.7)$$

and

$$\left(\frac{X_1}{X_3}\right)^{\beta_1}\left(\frac{\eta\left((\beta_1-1)Q_1Q_3\frac{\partial X_3}{\partial Q_1}-Q_1X_3\frac{\partial Q_3}{\partial Q_1}-Q_1Q_3\right)}{r-\mu}\right)$$

[2] Notice that this value is equal to $W_3(X,Q_1,Q_2,Q_3)$ at preemption trigger X_2, the value of waiting for being the third investor, i.e. $W_3(X_2,Q_1,Q_2,Q_3) = W_2(X_2,Q_1,Q_2,Q_3) = V_2(X_2,Q_1,Q_2,Q_3)$.

$$+\left(\frac{X_1}{X_2}\right)^{\beta_1}\left(\frac{\eta\left((\beta_1-1)Q_1Q_2\frac{\partial X_2}{\partial Q_1}-Q_1X_2\frac{\partial Q_2}{\partial Q_1}-Q_1Q_3\right)}{r-\mu}\right)$$
$$+\frac{X_1(1-2\eta Q_1)}{r-\mu}-c=0. \quad (4.3.8)$$

Additive demand 4.3.2

For additive demand, the value functions are constructed in a similar way as described for the multiplicative demand in Subsection 4.3.1. Here, we shall shortly summarize the firms' value functions and its optimal investment trigger or the implicit equation that has to be solved in order to find the optimal pre-emption moment.

The third investor's value function, $V_3(X,Q_1,Q_2,Q_3)$, is given by:

$$V_3(X,Q_1,Q_2,Q_3)=\begin{cases}(\frac{X}{X_3})^{\beta_1}(\frac{X_3Q_3}{r-\mu}-\frac{\eta(Q_1+Q_2+Q_3)Q_3}{r}-cQ_3) & X<X_3,\\ \frac{XQ_3}{r-\mu}-\frac{\eta(Q_1+Q_2+Q_3)Q_3}{r}-cQ_3 & X\geq X_3.\end{cases} \quad (4.3.9)$$

The optimal investment decision of the entrant is expressed in Proposition 4.4.

Proposition 4.4
Consider additive demand. The third investor invests at optimal investment trigger X_3, given by:

$$X_3=\frac{\beta_1(r-\mu)(cr+(Q_1+Q_2+Q_3)\eta)}{(\beta_1-1)r}, \quad (4.3.10)$$

and invests in the optimal level of capacity

$$Q_3=\frac{cr+(Q_1+Q_2)\eta}{(\beta_1-2)\eta}.$$

The value function of the second investor, $V_2(X,Q_1,Q_2,Q_3)$, is given by:

$$V_2(X,Q_1,Q_2,Q_3)=$$
$$\begin{cases}(\frac{X}{X_3})^{\beta_1}(-\frac{\eta Q_2Q_3}{r})+\frac{XQ_2}{r-\mu}-\frac{\eta(Q_1+Q_2)Q_2}{r}-cQ_2 & X<X_3,\\ \frac{XQ_2}{r-\mu}-\frac{\eta(Q_1+Q_2+Q_3)Q_2}{r}-cQ_2 & X\geq X_3.\end{cases} \quad (4.3.11)$$

The following proposition states the two equations that lead to the optimal investment decision of the second investor under additive demand.

Proposition 4.5
Consider additive demand. The preemption trigger X_2 and optimal capacity level Q_2 of the second investor are implicitly defined by

$$\left(\frac{X_2}{X_3}\right)^{\beta_1}\left(\frac{X_3Q_3}{r-\mu}-\frac{\eta(Q_1+Q_3)Q_3}{r}-cQ_3\right) - \frac{X_2Q_2}{r-\mu}+\frac{\eta(Q_1+Q_2)Q_2}{r}+cQ_2=0 \quad (4.3.12)$$

and

$$\left(\frac{X_2}{X_3}\right)^{\beta_1}\left(\frac{cr+(Q_1+Q_2)\eta-\eta Q_2\beta_1}{2-\beta_1 r}\right)+\frac{X_2}{r-\mu}-\frac{\eta(Q_1+Q_2)}{r}-c=0.$$

Finally, we analyze the investment decision of the first investor. The first firm's value function, $V_1(X,Q_1,Q_2,Q_3)$ is expressed by:

$$V_1(X,Q_1,Q_2,Q_3)=\begin{cases} (\frac{X}{X_3})^{\beta_1}(-\frac{\eta Q_1Q_3}{r})+(\frac{X}{X_2})^{\beta_1}(-\frac{\eta Q_1Q_2}{r}) \\ \qquad +\frac{XQ_1}{r-\mu}-\frac{\eta(Q_1)Q_1}{r}-cQ_1 & X<X_2, \\ (\frac{X}{X_2})^{\beta_1}(-\frac{\eta Q_2Q_3}{r})+\frac{XQ_1}{r-\mu}-\frac{\eta(Q_1+Q_2)Q_1}{r}-cQ_1 & X_2<X<X_3, \\ \frac{XQ_1}{r-\mu}-\frac{\eta(Q_1+Q_2+Q_3)}{r}-cQ_1 & X\geq X_3. \end{cases} \quad (4.3.13)$$

The first entrant needs to prevent that it is preempted by one of the other two potential entrants. Therefore it will invest when it is indifferent between waiting for the second investment position, or investing immediately, this preemption moment is denoted by X_1. The following proposition states the two equations that need to be solved in order to find the optimal investment decision of the market leader.

Proposition 4.6
Consider additive demand. The preemption trigger X_1 and optimal capacity level Q_1 of the second investor are implicitly defined by

$$\left(\frac{X_1}{X_3}\right)^{\beta_1}\left(\frac{\eta Q_3(Q_1-Q_2)}{r}\right)+\left(\frac{X_1}{X_2}\right)^{\beta_1}\left(\frac{X_2Q_2}{r-\mu}-\frac{\eta Q_2^2}{r}-cQ_2\right)$$

$$-\frac{X_1 Q_1}{r-\mu}+\frac{\eta Q_1^2}{r}+cQ_1=0 \quad (4.3.14)$$

and

$$\left(\frac{X_1}{X_3}\right)^{\beta_1}\left(\frac{\eta\left(\frac{\beta_1 Q_1 Q_3}{X_3}-Q_1\frac{\partial Q_3}{\partial Q_1}-Q_3\right)}{r}\right) + \left(\frac{X_1}{X_2}\right)^{\beta_1}\left(\frac{\eta\left(\frac{\beta_1 Q_1 Q_2}{X_2}-Q_1\frac{\partial Q_2}{\partial Q_1}-Q_2\right)}{r}\right)+\frac{X_1}{r-\mu}-\frac{2\eta Q_1}{r}-c=0.$$

Results 4.4

Multiplicative demand 4.4.1

This section considers multiplicative demand. The previous section showed that in order to find a firm's optimal investment decision, two equations have to be solved simultaneously. Often, it is only possible to solve these equations numerically (e.g. Huisman and Kort (2014), Boonman and Hagspiel (2014)). However, this section shows that analytical solutions exist for the optimal investment decisions. The following proposition gives an expression for each investment decision. It turns out that the investment decisions depend on four expressions (i.e. $\chi_2(\beta_1)$, $\chi_1(\beta_1)$, $\varphi_2(\beta_1)$ and $\varphi_1(\beta_1)$), that satisfy a system of four implicit equations. However, notice that these equations are only a function of β_1.

The following proposition also proves that the optimal investment moment is not affected by a change in substitutability parameter η. This parameter will only affect the optimal capacity, i.e. a higher level of substitutability leads to a strictly lower optimal capacity choice for each firm. This is intuitive, since a higher substitutability makes the price more sensitive to a change in the level of production (thus, the level of capacity). Therefore, the firm tries to keep the price high by investing in less capacity when the substitutability would be slightly higher. This explains why a firm rather uses its capacity choice as a tool to deal with a higher level of substitutability, than the timing of investment.

Furthermore, we find that the optimal capacities are not dependent on the unit cost of capacity, and the optimal investment trigger increases in the unit cost of capacity. That is, under the multiplicative demand structure, a firm reacts on a higher investment costs by delaying its investment. The multiplicative demand structure assumes a fixed market size and investing later does therefore not imply an increase in the size of the market.

Proposition 4.7

Consider the multiplicative demand structure. We find that the optimal investment decision of the first investor is equal to

$$X_1 = \frac{c(\beta_1+1)(r-\mu)}{\beta_1 - 1}\chi_1(\beta_1), \tag{4.4.1}$$

$$Q_1 = \frac{\varphi_1(\beta_1)}{(1+\beta_1)\eta}. \tag{4.4.2}$$

The optimal investment decision of the second investor is given by

$$X_2 = \frac{c(\beta_1-1)^2(r-\mu)\chi_2(\beta_1)}{(\beta_1-1)(1-\varphi_1(\beta_1)+\beta_1)}, \tag{4.4.3}$$

$$Q_2 = \frac{(1-\varphi_1+\beta_1)\varphi_2(\beta_1)}{(\beta_1+1)^2\eta}, \tag{4.4.4}$$

and for the third investor we have

$$X_3 = \frac{c(\beta_1-1)^3(r-\mu)}{(\beta_1-1)(1-\varphi_1(\beta_1)+\beta_1)(1-\varphi_2(\beta_1)+\beta_1)}, \tag{4.4.5}$$

$$Q_1 = \frac{(1-\varphi_1(\beta_1)+\beta_1)(1-\varphi_2(\beta_1)+\beta_1)}{(\beta_1+1)^3\eta}. \tag{4.4.6}$$

The expressions $\varphi_1(\beta_1)$, $\varphi_2(\beta_1)$, $\chi_1(\beta_1)$ and $\chi_2(\beta_1)$ can be found by simultaneously solving the following four equations:

$$\frac{(\varphi_2-1)(1+\beta_1)\left(\chi_2-\frac{\varphi_2\chi_2}{1+\beta_1}\right)^{\beta_1}}{1-\varphi_2+\beta_1}+1+\beta_1+\chi_2(1-2\varphi_2+\beta_1)=0, \quad (4.4.7)$$

$$\frac{(1+(1+\varphi_2)\beta_1)\left(\chi_2-\frac{\varphi_2\chi_2}{1+\beta_1}\right)^{\beta_1}}{(1+\beta_1)\varphi_2}+1+\beta_1+\chi_2(1-\varphi_2+\beta_1)=0, \quad (4.4.8)$$

$$\begin{aligned}&\left(\frac{\chi_1(1-\varphi_1+\beta_1)(1-\varphi_2+\beta_1)}{(1+\beta_1)^2}\right)^{\beta_1}(\varphi_1(\beta_1+1)+\beta_1-1)\\&+\left(\frac{\chi_1-\frac{\varphi_1\chi_1}{1+\beta_1}}{\chi_2}\right)^{\beta_1}(\beta_1+1)\chi_2\varphi_2(\varphi_1-1)\\&\qquad+(1-\varphi_1+\beta_1)(\varphi_1(1-2\varphi_1+\beta_1)+1-\beta_1)=0, \quad (4.4.9)\end{aligned}$$

and

$$\begin{aligned}&\left(\frac{\chi_1(1-\varphi_1+\beta_1)(1-\varphi_2+\beta_1)}{(1+\beta_1)^2}\right)^{\beta_1}\left(\frac{(\varphi_1-\varphi_2)(\beta_1+1)+\varphi_1\varphi_2}{\beta_1+1}\right)+\\&\left(\frac{\chi_1-\frac{\varphi_1\chi_1}{1+\beta_1}}{\chi_2}\right)^{\beta_1}\left(\frac{(\varphi_1-1+\chi_2)(\beta_1-1)+\chi_2(2+\varphi_1\varphi_2-\varphi_2(1+\beta_1))}{\beta_1+1}\right)\varphi_2\\&\qquad+\varphi_1((\beta_1-1)-\chi_1(1-\varphi_1-\beta_1))=0. \quad (4.4.10)\end{aligned}$$

The investment triggers X_1, X_2, and X_3 are not dependent of the substitutability parameter η and the optimal capacities Q_1, Q_2, Q_3 are not dependent of the unit cost of capacity c. Furthermore, there is a negative relationship between the optimal capacities and η, and a positive relationship between the optimal investment triggers and the unit cost of capacity.

Based on the expressions in Proposition 4.7 we cannot draw any conclusions about the effect of uncertainty on the investment decisions. Namely, $\chi_2(\beta_1)$, $\chi_1(\beta_1)$, $\varphi_2(\beta_1)$ and $\varphi_1(\beta_1)$ still depend upon β_1, which in turn is a function of market uncertainty. The left graph of Figure 4.1 shows the effect of the market uncertainty on the firm's investment trigger. The graph on the right hand side illustrates the corresponding optimal capacities. The effect of market

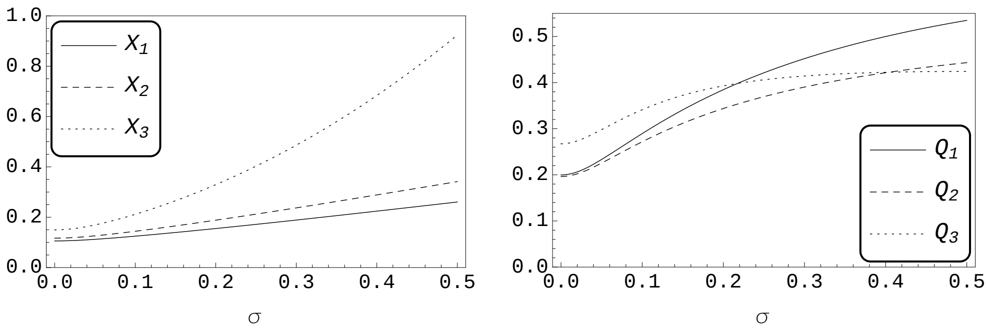

Figure 4.1: The effect of σ on the firm's optimal investment trigger and capacities, when multiplicative demand is considered. Take r=0.1, μ =0.01, η =0.5, and c =1.

uncertainty on the optimal capacities is interesting. Boonman and Hagspiel (2014) showed already for multiplicative demand, when there are just two firms in the market, that the second investor invests in a larger capacity than the first investor when market uncertainty is low. The opposite situation will occur when market uncertainty is high enough. In a market with three investors, we find that there are three possible scenarios.

The firms preempt each other for the first two market positions, and therefore they invest sooner than they would have done without the game-aspect. Investing soon implies that the market is not yet big enough for a large investment. Due to the structure of the multiplicative demand, a higher level of uncertainty parameter X increases the price. Due to the Stackelberg advantage of the first entrant, it can invest in a larger capacity than the second entrant and thereby occupy a bigger part of the market. When there is just one entrant left to invest, nobody can preempt this entrant anymore. Consequently, the third entrant can invest at its non-strategic optimal investment trigger. The capacity size of the third entrant, relative to the size of the first and second entrant, depends on the market uncertainty. For low uncertainty, the third entrant invests in the largest capacity. The other two entrants are forced to invest so soon, due to the preemption threat, that they are not able to invest in a larger capacity size. For a somewhat more uncertain market environment, the value of waiting will go up. This delays the preemption moments of the first and the second entrant, and enables them to invest in a larger capacity. Recall that, for multiplicative demand, the first entrant's capacity is always bigger than the capacity of the second entrant, i.e. the Stackelberg advantage. Therefore, for an intermediate market uncertainty level, the first entrant will have a larger capacity than the third entrant, and the second entrant will still have the smallest capacity size. The third scenario occurs for a highly uncertain market environment. In this case, also the preemption moment of the second entrant is delayed enough to obtain a larger capacity than the third entrant.

The left hand side of Figure 4.1 shows, for a specific market scenario, that $X_2 - X_1$ and $X_3 - X_2$ are increasing with uncertainty (σ). That is, the accordion effect does not occur. The fact that the optimal capacities and preemption triggers are still a function of parameters that can only be obtained by solving four implicit equations, makes it impossible to analytically prove that this is a general result. However, extensive numerical analysis shows that the gap between the first and the second, and the second and the third entrant increases with uncertainty, for all other parameter combinations.

Result 4.1
Consider the multiplicative demand structure and an oligopoly with three capacity optimizing firms. We find that the gap between any of the three investors increases for a higher market uncertainty. Consequently, neither a equilibrium where two or more firms invest at the same time, nor the accordion effect seem to occur.

Additive demand 4.4.2

The previous subsection gives an analysis of the investment decisions in case of multiplicative demand. In this subsection we consider additive demand. Again, it is possible to obtain analytical expression for the optimal capacities and investment triggers. These are still a function of four expressions in β_1, which can be found by solving four implicit equations. The analytical expressions show us that the investment moments of all firms are unaffected by substitutability parameter η, as was also derived for the multiplicative demand structure. Also, we find that the optimal capacity and optimal investment moments are linearly increasing in unit capacity cost c. Where a higher unit capacity cost is expected to have a diminishing effect on the optimal capacity choice, it also has an increasing effect on the timing of investment. A firm with the additive demand delays the moment of investment to an extend that it is also able to invest in a larger capacity size. Contrary to the multiplicative demand structure, that has a fixed market size, the additive demand structure allows for an increase in the size of the market when a firm delays its moment of investment. Therefore, under the additive demand structure an increase in unit capacity costs leads to a higher level of capacity, where it has no effect on the optimal level of capacity under multiplicative demand.

The expressions for the optimal investment triggers with the corresponding conclusions are formulated in Proposition 4.8.

Proposition 4.8

Consider the additive demand structure. We find that the optimal investment decisions of the first investor is equal to

$$X_1 = \frac{\beta_1 c(r-\mu)}{\beta_1 - 2}\chi_1(\beta_1),$$

$$Q_1 = \frac{cr}{\eta}\varphi_1(\beta_1).$$

The optimal investment decision of the second investor is given by

$$X_2 = \frac{\beta_1 c(r-\mu)}{\beta_1 - 2}(1+\varphi_1(\beta_1))\chi_2(\beta_1),$$

$$Q_2 = \frac{cr}{\eta}\frac{1+\varphi_1(\beta_1)}{\beta_1 - 2}\varphi_2(\beta_1),$$

and for the third investor we have

$$X_3 = \frac{\beta_1 c(r-\mu)}{\beta_1 - 2}(1+\varphi_1(\beta_1))(1+\frac{\varphi_2(\beta_1)}{\beta_1 - 2}),$$

$$Q_3 = \frac{cr}{\eta}\frac{1+\varphi_1(\beta_1)}{\beta_1 - 2}(1+\frac{\varphi_2(\beta_1)}{\beta_1 - 2}).$$

Expressions $\varphi_1(\beta_1)$, $\varphi_2(\beta_1)$, $\chi_1(\beta_1)$ and $\chi_2(\beta_1)$ can be found by simultaneously solving the following four equations:

$$\left(\chi_2 - \frac{\varphi_2\chi_2}{\varphi_2+\beta_1-1}\right)^{\beta_1}(\varphi_2-1) - 2(\varphi_2-1) + \beta_1(\chi_2-1) = 0$$

$$\left(\chi_2 - \frac{\varphi_2\chi_2}{\varphi_2+\beta_1-1}\right)^{\beta_1}\left(\frac{(\beta_1+\varphi_2-2)(\beta_1(\varphi_2+1)-\varphi_2-2)}{(\beta_1-2)^2}\right) + \varphi_2(\beta_1(1-\chi_2)+\varphi_2-2)=0$$

$$\left(\frac{\chi_1(\beta_1-2)}{(\varphi_2+\beta_1-2)(1+\varphi_1)}\right)^{\beta_1}\frac{(1+\varphi_1)(\varphi_2+\beta_1-2)(\varphi_1(1+\varphi_1)-\varphi_2(\beta_1-2))}{(\beta_1-2)^3}$$
$$+\left(\frac{\chi_1}{\chi_2(1+\varphi_1)}\right)^{\beta_1}\frac{((\chi_2\beta_1-\varphi_2)(\varphi+1)+\beta_1-2)(1+\varphi_1)\varphi_2}{(\beta_1-2)^2}$$
$$+\varphi_1\left(1+\varphi_1-\frac{\chi_1\beta_1}{\beta_1-2}\right)=0,$$

$$\left(\frac{\chi_1(\beta_1-2)}{(\varphi_2+\beta_1-2)(1+\varphi_1)}\right)^{\beta_1}(\varphi_2+\beta_1-2)(\varphi_1(\beta_1-2)-1)+$$
$$+\left(\frac{\chi_1}{\chi_2(1+\varphi_1)}\right)^{\beta_1}(\varphi_2(\beta_1-2))(\varphi_1(\beta_1-2)-1)$$
$$-(\beta_1-2)((2\chi_1+1)(\beta_1-2)-\chi_1\beta_1)=0.$$

The optimal investment moments X_1, X_2, and X_3 are not dependent of the substitutability parameter η, and the corresponding levels of capacity, Q_1, Q_2, and Q_3, are increasing in substitutability parameter η. The timing of investment and the optimal level of capacity are both linearly increasing in unit capacity cost c.

Since there are still four expressions that have to be solved by use of implicit equations, we cannot draw any conclusions on the effect of uncertainty, based on the expressions of Proposition 4.8. Figure 4.2 shows the effect of uncertainty on the optimal investment moments and corresponding optimal capacities, for a representative parameter scenario. We find that also for additive demand, due to the possibility to optimize capacities, the gap between all firms' investments increase with uncertainty.

Notice that it always holds that $Q_1 < Q_2 < Q_3$, contrary to the multiplicative demand, where there were three scenarios. The difference lies in the fact that a higher level of X increases the price for both the additive and multiplicative demand function, but, in addition it also increases the market size for the additive demand function. Waiting for a higher level of X leads to a bigger market size, in case of additive demand, which gives an entrant the opportunity

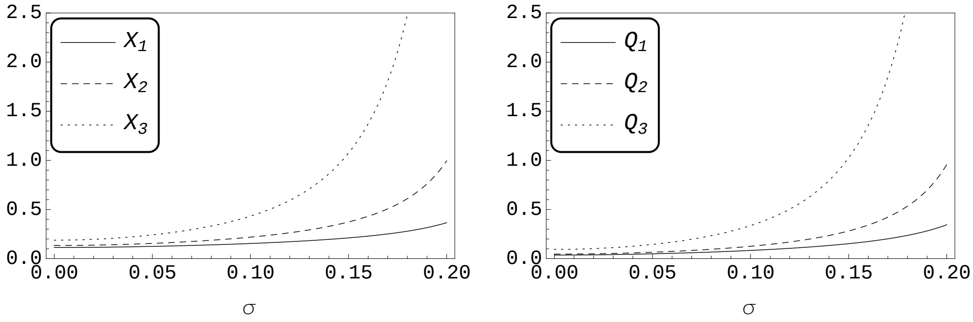

Figure 4.2: The effect of σ on the firm's optimal investment triggers and capacities, when additive demand is considered. Take r=0.1, μ =0.01, η =0.5, and c =1.

to invest in a larger capacity than the previous entrant. Manne (1961) is an early contribution that finds, for a monopolistic firm, that it invests in a larger capacity level when uncertainty increases (size effect).

The accordion effect, found by Bouis et al. (2009) indicates that the preemption moment of the second investor moves towards the preemption moment of the first investment, and the investment moment of the third investor increases for a higher level of market uncertainty. Figure 4.2 illustrates that allowing firms to optimize capacities makes them all move in the same direction, and consequently the size effect dominates the accordion effect. Also for the additive demand, we cannot show analytically that the accordion effect does not occur under capacity optimization, but our extensive numerical analysis show that this result holds in general.

Result 4.2
Consider the additive demand structure and an oligopoly with three firms where each firm optimizes capacity. We find that the gap between any of the three investors increases for a higher market uncertainty. Consequently, neither the simultaneous equilibrium nor the accordion effect seem to occur.

4.5 Conclusions

This chapter analyzes the optimal investment timing and capacity choice of three potential entrants, where firm roles are assumed to be endogenously determined. We examine how resistent the result of Bouis et al. (2009) is under the assumption of different demand structures (i.e. multiplicative and additive demand) and the relaxation of the assumption of given capacity. Bouis et al. (2009) analyzes optimal timing of investment for three firms under the assumption of a simplified multiplicative demand structure. They find that a higher level of market uncertainty increases the third firm's threshold, decreases the second firm's threshold and increases the first firm's threshold. Hence, the wedge between the first and the second threshold decreases and the wedge between the second and the third threshold increases. This is defined as the accordion effect. For high enough uncertainty, it is optimal for the first two entrants to invest simultaneously.

Under capacity optimization, both types of demand functions show that all firms invest sequentially. Furthermore, the first and the second firm never invest in the same capacity. Under additive demand, later investors invest in a larger capacity. This is due to a feature of the demand structure, which allows for an increase in the market size. Under multiplicative demand however, the relative investment magnitude of the three investors is dependent of uncertainty, as is also explained by Boonman and Hagspiel (2014) for the duopoly case.

Another point where this chapter contributes to the existing literature, is that we are able to show analytically that increasing substitutability has a decreasing effect on the capacity choice, while it does not affect the timing of investment. Furthermore, under the assumption of additive demand it can be shown that higher investment costs leads to a later investment in a larger capacity, where under the assumption of multiplicative demand this leads to a later investment, but the optimal capacity choice of the firm is unaffected. The additive demand structure allows for an increase in the market size when investment is delayed, which stimulates a firm to invest in a larger capacity in case it delays investment. The market size under multiplicative demand however, is fixed for all investment moments.

Where Bouis et al. (2009) also consider the multi-firm case, this chapter is limited to the three-firm case. It is most likely that our results can be generalized for the multi-firm case. However, this remains to be investigated in future research. Also, for analytical convenience we proposed the market clearance assumption. However, it would be interesting to analyze how our results change under production flexibility.

4.A Appendix

4.A.1 Proof of Proposition 4.1

The value function of the third investor is given by:

$$V_3(X,Q_1,Q_2,Q_3) = \mathbb{E}\left(\int_{t=0}^{\infty} e^{-rt}X_t(1-\eta(Q_1+Q_2+Q_3))Q_3\,\mathrm{d}t - cQ_3\right)$$
$$= \frac{XQ_3(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_3.$$

The value of the firm before investment, i.e. the value of waiting, takes the form $f(X) = AX^{\beta_1}$, where A is a constant to be determined and β_1 is the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta_1^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta_1 - r = 0.$$

Denote with X_3 the investment moment of the follower. Value matching and smooth pasting the follower's value of waiting with the value function after investment results in solving the following two equations:

$$AX_3^{\beta_1} = \frac{X_3Q_3(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_3,$$

$$\beta_1 AX_3^{\beta_1-1} = \frac{Q_3(1-\eta(Q_1+Q_2+Q_3))}{r-\mu}.$$

Additionally, the optimal capacity of the third entrant is derived by maximizing the value function with respect to Q_3 and solving the first order partial derivative condition:

$$\frac{X_3\,(1-\eta(Q_1+Q_2+2Q_3))}{r-\mu} - c = 0.$$

Therefore,

$$X_3 = \frac{c(\beta_1+1)(r-\mu)}{(\beta_1-1)(1-\eta(Q_1+Q_2+Q_3))},$$

and

$$Q_3 = \frac{c(1-\eta(Q_1+Q_2))}{\eta(\beta_1^2-1)}.$$

□

Proof of Proposition 4.2 4.A.2

The second investor invests at preemption moment X_2, where $W_3(X_2,Q_1,Q_2,Q_3) = V_2(X_2,Q_1,Q_2,Q_3)$. Here, $W_3(X,Q_1,Q_2,Q_3)$ is given by

$$W_3(X,Q_1,Q_2,Q_3) = \left(\frac{X}{X_3}\right)^{\beta_1}\left(\frac{X_3Q_3(1-\eta(Q_1+Q_2+Q_3))}{r-\mu} - cQ_3\right)$$

and $V_2(X,Q_1,Q_2,Q_3)$ is given by

$$V_2(X,Q_1,Q_2,Q_3) = \left(\frac{X}{X_3}\right)^{\beta_1}\left(-\frac{\eta Q_2Q_3X_3}{r-\mu}\right) + \frac{XQ_2(1-\eta(Q_1+Q_2))}{r-\mu} - cQ_2. \tag{4.A.1}$$

$W_3(X_2,Q_1,Q_2,Q_3) - V_2(X_2,Q_1,Q_2,Q_3) = 0$ is equivalent to expression (4.3.4). Expression (4.3.5) in Proposition 4.2 is found by taking the first order partial derivative of $V_2(X,Q_1,Q_2,Q_3)$ with respect to Q_2. □

Proof of Proposition 4.3 4.A.3

The first investor invests at preemption moment X_1, where $W_2(X_1,Q_1,Q_2,Q_3) = V_1(X_1,Q_1,Q_2,Q_3)$. Here, $W_2(X,Q_1,Q_2,Q_3)$ is given by

$$W_2(X,Q_1,Q_2,Q_3) = \left(\frac{X}{X_2}\right)^{\beta_1}\left(\left(\frac{X_2}{X_3}\right)^{\beta_1}\left(-\frac{\eta Q_2Q_3X_3}{r-\mu}\right) + \frac{X_2Q_2(1-\eta(Q_1+Q_2))}{r-\mu} - cQ_2\right)$$

and $V_2(X,Q_1,Q_2,Q_3)$ is given by

$$V_1(X,Q_1,Q_2,Q_3) =$$

$$\left(\frac{X}{X_3}\right)^{\beta_1}(-\frac{\eta Q_1 Q_3 X_3}{r-\mu})+\left(\frac{X}{X_2}\right)^{\beta_1}(-\frac{\eta Q_1 Q_2 X_2}{r-\mu})+\frac{XQ_1(1-Q_1\eta)}{r-\mu}-cQ_1. \tag{4.A.2}$$

$W_2(X_1,Q_1,Q_2,Q_3)-V_1(X_1,Q_1,Q_2,Q_3)=0$ is equivalent to expression (4.3.7). Expression (4.3.8) in Proposition 4.3 is found by taking the first order partial derivative of $V_1(X,Q_1,Q_2,Q_3)$ with respect to Q_1. □

4.A.4 Proof of Proposition 4.4

The concept of this proof is analogous to the proof of Proposition 4.1.

4.A.5 Proof of Proposition 4.5

The concept of this proof is analogous to the proof of Proposition 4.2.

4.A.6 Proof of Proposition 4.6

The concept of this proof is analogous to the proof of Proposition 4.3.

4.A.7 Proof of Proposition 4.7

In Proposition 4.7 we find analytical solutions for the investment triggers of the three firms. For the last firm, we are able to find analytical solutions for X_3 and Q_3. Namely, we simultaneously solve the value matching and smooth pasting conditions in the proof of Proposition 4.1 and the equation that gives the first order partial derivative of the $V_3(X_3,Q_1,Q_2,Q_3)$ with respect to Q_3. We obtain:

$$X_3=\frac{c(\beta_1+1)(r-\mu)}{(\beta_1-1)(1-\eta(Q_1+Q_2))}, \tag{4.A.3}$$

$$Q_3=\frac{1-\eta(Q_1+Q_2)}{(\beta_1+1)\eta}. \tag{4.A.4}$$

When we use these solutions we find

$$X_2 = \frac{c(\beta_1+1)(r-\mu)}{(\beta_1-1)(1-\eta(Q_1))}\chi_2(\beta_1), \tag{4.A.5}$$

$$Q_2 = \frac{1-\eta Q_1}{(\beta_1+1)\eta}\varphi_2(\beta_1) \tag{4.A.6}$$

for some $\chi_2(\beta_1)$ and $\varphi_2(\beta_1)$. Similarly, we find

$$X_1 = \frac{c(\beta_1+1)(r-\mu)}{\beta_1-1}\chi_1(\beta_1), \tag{4.A.7}$$

$$Q_1 = \frac{1}{(\beta_1+1)\eta}\varphi_1(\beta_1) \tag{4.A.8}$$

for some $\chi_1(\beta_1)$ and $\varphi_1(\beta_1)$. Substituting the latter two expressions expressions into the first four expressions gives the analytical solutions stated in Proposition 4.7. The analytical solutions need to satisfy four equations, namely the rent equalization formulas (given by equation (4.3.4) and (4.3.7)), and the first order conditions of the value functions which are given by:

$$-\frac{c^2\left(\frac{X_2(1-\eta(Q_1+Q_2))(\beta_1-1)}{c(r-\mu)(\beta_1+1)}\right)^{\beta_1}(1-\eta(Q_1+Q_2)+Q_2\eta\beta_1)}{(1-\eta(Q_1+Q_2))(1-\beta_1)^2} + \frac{X_2(1-\eta(Q_1+Q_2))}{r-\mu} - c = 0$$

and

$$\frac{-\eta\left(\frac{X_1}{X_3}\right)^{\beta_1}Q_1\left(X_2\frac{\partial Q_2}{\partial Q_1}+X_3\frac{\partial Q_3}{\partial Q_1}\right)}{r-\mu} + \frac{-\eta\left(\frac{X_1}{X_3}\right)^{\beta_1}Q_2\left(X_2-Q_1(\beta_1-1)\frac{\partial X_2}{\partial Q_1}\right)}{r-\mu} + \frac{-\eta\left(\frac{X_1}{X_3}\right)^{\beta_1}Q_3\left(X_3Q_1(\beta_1-1)\frac{\partial X_3}{\partial Q_1}\right)}{r-\mu} + \frac{X_1(1-2\eta Q_1)}{r-\mu} - c = 0,$$

respectively. Substitution of (4.4.1), (4.4.2), (4.4.3), (4.4.4), (4.4.5) and (4.4.6) into these four equations, gives the implicit equations (4.4.7), (4.4.8) and (4.4.9) and (4.4.10) that remain to be solved for $\chi_1(\beta_1)$, $\varphi_1(\beta_1)$, $\chi_2(\beta_1)$ and $\varphi_2(\beta_1)$ and are stated in Proposition 4.7. □

4.A.8 Proof of Proposition 4.8

The concept of this proof is analogous to the proof of Proposition 4.7.

5 Capacity Optimization in an Operational Model with Time-Lags

This chapter is based on Boonman and Siddiqui (2014).

Abstract

In this chapter, we consider a firm that has the opportunity to suspend and resume production infinitely many times. We contribute to the literature by allowing a firm to optimize the level of capacity prior to its suspension options and by incorporating a time lag in which a firm prepares the start of the production process. This time lag takes place after the decision to resume production. We find that an increase in the length of the time lag results in an increase in the optimal capacity level. Capacity optimization also determines how the length of the time lag affects the optimal investment and operational triggers. Under the assumption of a fixed level of capacity the length of the time lag lowers the operational triggers, while under capacity optimization these triggers can also increase. Namely, under capacity optimization, a larger time lag results in a larger capacity choice, which can indirectly result in higher operational triggers and a higher investment trigger. This indirect effect dominates when the level of market uncertainty is low and the initial time lag is small.

5.1 Introduction

Much of the existing literature in the real options field assumes that after investment a firm will always remain open, regardless of the current market price or demand (e.g. McDonald and Siegel (1986b), Dixit (1991) and Bouis et al. (2009)). However, given the volatile nature of deregulated industries, it would be plausible to assess a firm's desire to suspend the facility. With the recession that started in the fall of 2008, the flexibility of such firms to respond to market conditions is paramount. For example, in December 2008, General Motors announced that it would "Temporarily close twenty factories across North America and make sweeping cuts to its vehicle production as it tries to adjust to dramatically weaker automobile demand"[1]. Other car manufacturers, such as Toyota[2] and Honda[3], were forced to close some of their production plants. In turn, the global steel market, which also experienced a major cutback in steel purchases from its customers (e.g. automotive manufacturers), was also affected by the global economic downturn. To illustrate, in May 2009 the world largest steel producer ArcelorMittal was forced to idle its Monessen coke plant[4]. In the years following, the steel makers were battling back from the slump that begun when the economy faltered in 2008. Eventually, in 2012 ArcelorMittal announced to resume the production at its Monessen coke plant. Only two years later the production was ready to take off. Such production facilities are characterized by some relevant features. That is, once decided to resume operations, the firm faces a time lag to prepare the production process for the ultimate production. Also, there are some fixed costs associated with switching between the operational and the suspension state. These features should be taken into account when analyzing the optimal timing to resume and suspend the production in a plant.

Given this background, we assess the problem of a firm with the opportunity to suspend and resume production infinitely many times in the future against some positive switching costs (i.e. the mothballing option). Entry and exit models have been pioneered by Mossin (1968) and generalized by Brennan and

[1] http://articles.economictimes.indiatimes.com/2008-12-13/news/28479230_1_tony-sapienza-gm-and-chrysler-vehicle-production

[2] http://content.time.com/time/business/article/0,8599,1919395,00.html

[3] http://www.theguardian.com/business/2009/jan/30/honda-swindon-shutdown

[4] http://www.post-gazette.com/business/2014/04/08/Monessen-coke-plant-set-to-resume-production-this-month/stories/201404080025

Schwartz (1985), Dixit (1989), and McDonald (2002). Where these models place their focus on the optimal triggers for switching towards the other state, some later extensions take up the additional challenge to optimize endogenously the firm's capacity. Van Mieghem and Dada (1999) consider a firm that optimizes capacity before demand has been resolved. After demand realization, the firm chooses optimal production quantities, which are constraint by the earlier chosen capacity size. Dangl (1999) and Hagspiel (2011) extend this model into a dynamic setting. The continuous time framework allows a firm to optimize timing of investment along with capacity size. After investment, the firm optimizes the level of production for each realization of demand, assuming the same constraints as in Van Mieghem and Dada (1999).

The latter three models provide a contribution to the existing literature by optimizing the level of capacity. However, they allow a firm to switch costlessly between the operational and suspension states. In Dixit (1989) and McDonald (2002), a fixed cost has to be paid for switching between these two states, which brings the model closer to reality. Laying off employees or depreciation of unused equipment results in (insuperable) costs when a firm decides to temporarily close down its business. Furthermore, resuming production generates costs like overdue maintenance, marketing, and training that comes with hiring new employees. In case of the steel producer example, ArcelorMittal even reinvested $50 million in the plant prior to the reopening.

We use the McDonald (2002) model as the baseline model in our paper and make two contributions to it. First, we introduce an inverse demand structure, which captures the tradeoff between high capacity and low price in terms of the revenue. The firm has a one-time opportunity to make a lumpy capacity investment. This extension comes close to the mothballing models, Dangl (1999) and Hagspiel (2011), but with additional switching costs. Second, we introduce a time lag to prepare the production process, which takes place after the decision to resume production. Even for modest time lags there still is a significant effect on the optimal triggers to switch from the operational to the idle state and vice versa. Namely, this time lag will occur repeatedly after every decision of the production facility to resume production. In related work, Bar-Ilan and Strange (1996) embed lags in the classic irreversible lumpy investment model presented by Dixit (1989). They find that for some parameter values, an increase in uncertainty can actually hasten investment, a result contrary to that found in papers without investment lags. We, thus, incorporate a time lag in McDonald (2002)'s entry and exit model after each decision to resume production, since a start-up will not occur instantaneously. While the time it takes to prepare the production process is captured within this time lag, the decision to stop the production is assumed to be executed immediately. For example, in case of ArcelorMittal, it took two years between the announcement of the steel producer to resume production of the Monessen coke plant and the actual restart of the production.

The paper most closely related to our work is Sødal (2006), who uses the Dixit (1989) model as a baseline. In Sødal (2006), a firm has infinitely many options to enter and exit a market against a fixed entry or exit cost. Similar to our result, he finds that due to the fixed operational costs, a firm should wait for a strictly higher trigger price than the Marshallian cost. The effect of a time lag in resuming production implies that increasing uncertainty might hasten investment when there is a time lag and also lower the entry trigger for resuming operations. However, these outcomes are not as likely as argued in Bar-Ilan and Strange (1996). We deviate from the model of Sødal (2006) by allowing for capacity optimization. Furthermore, we assume that a firm only once decides about the level of its capacity, at the initial investment moment. In the operational decisions, following upon this, the firm is limited to this initial capacity choice.

The introduction of capacity optimization in the flexible operational models with a time lag changes the results of Bar-Ilan and Strange (1996) and Sødal (2006). Under a fixed capacity level, the effect of uncertainty on the optimal operational triggers is ambiguous. However, under capacity optimization, the level of capacity dominates the switching triggers. Namely, we find that a higher level of uncertainty leads to a delay in investment, which allows for a larger capacity level, and in turn leads to strictly higher switching triggers. The operational switching triggers occur around the level of price intercept for which the price is equal to the unit production cost. A higher capacity level requires a higher price intercept (i.e. trigger) to prevent prices from turning negative. Furthermore, capacity optimization changes the effect of the length of the time lag on the optimal investment trigger and the optimal switching triggers. Where Bar-Ilan and Strange (1996) observe that the trigger values decrease for an increase in the length of the time lag, we find that for low uncertainty and a low initial time lag the triggers can also increase. That is, a larger time lag results in a larger capacity choice, which in turn increases the trigger values. The larger capacity choice is a result of a truncated downside of investment and unlimited upward potential. Thus, at the moment that the firm pursues the production, after the time lag, it expects the profit to have grown. This result will be amplified under a larger time lag, thereby allowing for a larger capacity choice.

This chapter is structured as follows. The general model is presented in Section 5.2. Section 5.3 analyzes the model where a firm optimizes capacity at the moment of investment. After investment, it has infinitely many options to suspend and resume the production facility against a fixed switching cost. In Section 5.4, a time lag after the decision to resume production is added to the model. Section 5.5 compares the results for the model with and without a time lag, and Section 5.6 concludes.

Modeling Assumptions 5.2

Herein, we take the perspective a firm that has to decide about capacity investment in a new market. This involves two decisions, viz. when to invest and determining the size of capacity. After investment, the firm is in production. Against switching cost S_s it can decide to temporarily stop the production. In this state, the profit of the firm is zero. An inactive firm has the option to restart the production against switching cost S_r. Due to the (positive) switching costs, a firm will not switch at the price where its profit is equal to zero. Instead, it converts at some optimal switching triggers, i.e., at some uncertainty level it switches from the suspension state towards the operational state and at some other uncertainty level it switches back to the suspension state.

We assume that the inverse demand function is given by:

$$P_t = X_t - \eta Q, \tag{5.2.1}$$

where Q is the fixed annual production of the firm and η the substitutability parameter with $\eta \in (0,1)$. The uncertainty parameter X_t is uncertain and is assumed to follow the geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t d\omega_t \qquad t \geq 0,$$

$$X_0 = x,$$

with drift parameter μ, volatility parameter σ and $d\omega_t$ the increment of a standard Wiener process. Future revenues will be discounted against interest rate ρ. Variable production cost are denoted by c, so that the profit flow of a firm in production is given by:

$$\pi_t = (P_t - c)Q.$$

For analytical convenience, we propose the "market clearance assumption", i.e., the firm produces up to capacity. This assumption is widely used in the literature (Chod and Rudi (2005), Deneckere et al. (1997), Anand and Girotra (2007)). Unlike Dangl (1999) and Hagspiel (2011), we do not optimize a firm's production level; instead, the firm is able to deal with low prices via the option to suspend the production facility.

Note that we consider additive demand. This demand structure runs the risk to produce negative prices for very low levels of X_t. However, the firm has the option to suspend the production facility for low prices, i.e., for low levels of X_t. As a result, a firm never produces for those scenarios where prices are below zero. Following Hagspiel (2011) and Dangl (1999), investment costs are sunk and equal to $I(Q_t) = \delta Q_t^{\lambda}$. Constant $\delta > 0$ denotes the variable investment cost and constant $\lambda > 0$ is assumed to be less than one, thereby implying a concave investment structure. By adopting the same investment costs, in a later section we are able to make a comparison between these papers and our results[5].

[5]Hagspiel (2011) additionally shows the scenario where a firm faces convex investment costs. This paper explains that "the investment cost the firm is facing for the convex case is significantly higher for larger investments, and therefore installing a large amount of capacity is more expensive."

Operational Flexibility under Capacity Choice 5.3

To illustrate a sequence of decisions made by a firm after investment, consider Figure 5.1, which illustrates a possible path of the uncertainty parameter X_t. Assume that the considered firm is not producing at the start of the time frame. We find that the firm resumes the production at time $t = \tau_{2k}$. The firm stays in production for a while until it suspends production at time $t = \tau_{2k+1}$. After this closure decision, the firm resumes the production at time $t = \tau_{2k+2}$.

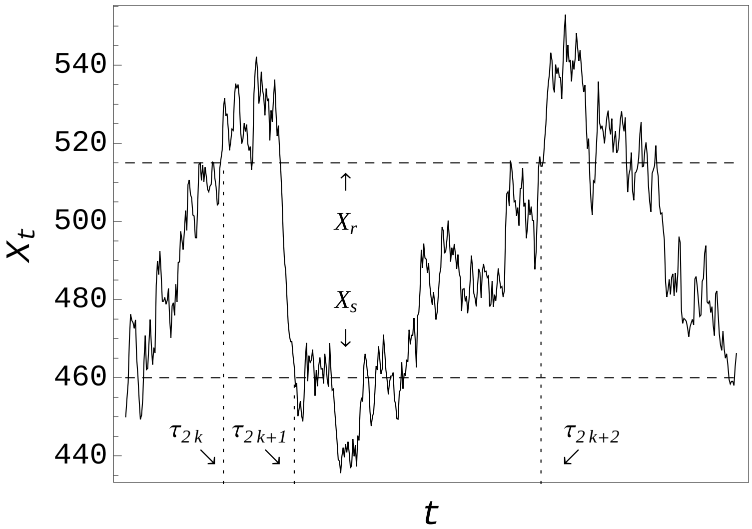

Figure 5.1: A simulated path of uncertainty parameter X_t, with operational triggers $X_s = 460$ and $X_r = 515$.

The Model 5.3.1

Figure 5.2 shows the decision-making time line. At time $t = \tau_I$ the firm invests in optimal capacity size Q^*. The firm suspends operations at the odd triggers and resumes at the even triggers, at time $t = \tau_{2k+1}$ and $t = \tau_{2k+2}$, respectively, for $k \in \mathbb{N}$, where $\tau_1 > \tau_I$. Since there exists stationary behavior in the shut down and re-starting decisions, we assume that there is one mutual trigger for resuming operations, i.e. $X_{\tau_{2k}} = X_r$ for all $k \in \mathbb{N}$, and one mutual trigger for suspending operations, i.e. $X_{\tau_{2k+1}} = X_s$, for all $k \in \mathbb{N}$. In order to find the value functions and optimal decisions of the firm, we will work backwards through the time line. Since a firm has infinitely many charging and discharging

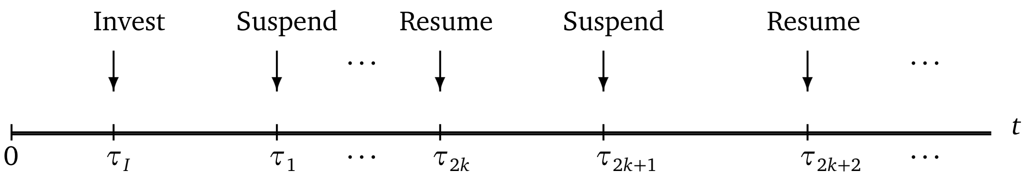

Figure 5.2: Decision-making time line for investing, suspending and resuming operations.

decisions, it is not possible to consider the “last" decision of a firm. Instead, we shall consider an arbitrary moment on the time line and work backwards from that moment on.

To start with, assume that at capacity size Q the firm has just suspended the production at X_s at time τ_{2k+1}, and it can now evaluate the decision to resume operations at level X_r at time τ_{2k+2}. The value of this firm is given by $F_s(X_t;X_s,X_r,Q)$. Here, X_t is the level of the uncertainty parameter at time t. Taking one step back in the time line, we look at the decision to suspend operations. Denote the net expected value of a plant that is in operation by $F_r(X_t;X_s,X_r,Q)$. X_t expresses the level of uncertainty at the time that a firm considers to suspend the production. Therefore:

$$
\begin{aligned}
F_r(X_t;X_s,X_r,Q) = {} & \frac{X_tQ}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho} + \\
& \left(\frac{X_t}{X_s}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho} - \frac{X_sQ}{\rho-\mu} - S_s + F_s(X_s;X_s,X_r,Q)\right). \quad (5.3.1)
\end{aligned}
$$

The first two terms in expression (5.3.1) represent the expected revenue of a firm that produces forever. At some point in the future, the firm decides to suspend operations and loses this revenue. Additionally, it incurs switching cost S_s, and receives option value $F_s(X_s;X_s,X_r,Q)$. The term $\left(\frac{X_t}{X_s}\right)^{\beta_2}$ can be interpreted as a stochastic discount factor. Dixit and Pindyck (1994) show that $\mathbb{E}[e^{-r(\tau_{2k+1}-t)}] = \left(\frac{X_t}{X_s}\right)^{\beta_2}$. Here, β_1 (β_2) is the positive (negative) root of the quadratic polynomial

$$
\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - \rho = 0.
$$

By taking one extra step back in the decision-making time line, we (again) look at the decision to resume operations, occurring at time $t = \tau_{2k}$. The net expected value of a plant that has just stopped the production and now has the option to resume is given by:

$$F_s(X_t;X_s,X_r,Q) = \left(\frac{X_t}{X_r}\right)^{\beta_1} (F_r(X_r;X_s,X_r,Q) - S_r). \tag{5.3.2}$$

Expression (5.3.2) explains that the firm obtains no profit in the suspension state, but it has an option to resume operation against switching cost S_r in the future.

After substitution of $F_s(X_s;X_s,X_r,Q)$ into $F_r(X_r;X_s,X_r,Q)$, we find that the solution for $F_r(X_r;X_s,X_r,Q)$. Since this solution is not equal to $F_r(X_r;X_s,X_r,Q)$ as defined in equation (5.3.1), we denote this solution by $G_r(X_s;X_r,Q)$.

$$G_r(X_s;X_r,Q) = \frac{\frac{X_rQ}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho} + \left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{X_s}{X_r}\right)^{\beta_1}(-S_r) + \left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho} - \frac{X_sQ}{\rho-\mu} - S_s\right)}{1-\left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{X_s}{X_r}\right)^{\beta_1}}. \tag{5.3.3}$$

Subsequently, we substitute equation (5.3.3) into $F_s(X_s;X_s,X_r,Q)$. The solution we denote by $G_s(X_r;X_s,Q)$:

$$G_s(X_r;X_s,Q) = \left(\frac{X_s}{X_r}\right)^{\beta_1} \left(\frac{\frac{X_sQ}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho} + \left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{X_s}{X_r}\right)^{\beta_1}(-S_r) + \left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho} - \frac{X_sQ}{\rho-\mu} - S_s\right)}{1-\left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{X_s}{X_r}\right)^{\beta_1}}\right).$$

We take the first-order necessary condition of $G_s(X_r;X_s,Q)$ with respect to X_r, and the first-order necessary condition of $G_r(X_s;X_r,Q)$ with respect to X_s. We simultaneously solve these two equations to find the two switching triggers $X_s(Q)$ and $X_r(Q)$.

The switching triggers are relevant only after the firm has performed investment. Now, assume a firm that has not yet invested in the market. Here, X_t is the level of the uncertainty parameter at the first moment where the firm considers investing in the market. The value of a firm that has the option to invest at $X_t = X_I$ in capacity Q is given by

$$
\begin{aligned}
&F_I(X_t; X_I, Q) = \\
&\left(\frac{X_t}{X_I}\right)^{\beta_1} \left(\frac{X_I Q}{\rho - \mu} - \frac{(\eta Q^2 + cQ)}{\rho} - \delta Q^{\lambda} + \left(\frac{X_I}{X_s(Q)}\right)^{\beta_2} \left(\frac{(\eta Q^2 + cQ)}{\rho}\right.\right. \\
&\qquad\qquad\qquad \left.\left. - \frac{X_s(Q)Q}{\rho - \mu} - S_s + F_s(X_s(Q); X_s(Q), X_r(Q), Q)\right)\right).
\end{aligned}
$$

We find the optimal moment to invest and the corresponding optimal level of capacity by optimizing $F_I(X_t; X_I, Q)$ with respect to X_I and Q, resulting in X_I^* and Q^*. Due to the complexity of the first order derivatives, there is no analytical solution available for the above stated problem. Bar-Ilan and Strange (1996) explain that under some simplifying assumptions, it is possible to find closed-form solutions for $X_s(Q)$ and $X_r(Q)$. In order to perform the sensitivity analysis with respect to certain parameters, we use the software program Mathematica.

5.3.2 Results

The upper panel of Figure 5.3 illustrates the effect of market uncertainty on the switching triggers X_s and X_r for a fixed level of capacity. Capacity choice Q is chosen optimal for σ =0.1. The result is in line with the standard real options result, viz. a higher level of uncertainty delays the firm's switching triggers. For a more uncertain market environment, the firm waits for a higher (lower) level of X_t before it switches from the suspended (operational) state towards the operational (suspended) state. The bottom panel of Figure 5.3 illustrates the effect of capacity choice Q on the optimal switching triggers. Due to the positive switching costs, the triggers to suspend and resume operations, respectively, lie below and above the level of X_t for which the profit is exactly zero. The inverse demand function, as defined by expression (5.2.1), shows that a higher capacity level implies a higher level of X_t in order to avoid a negative price. Therefore, the switching triggers increase for a higher capacity level as illustrated in Figure 5.3.

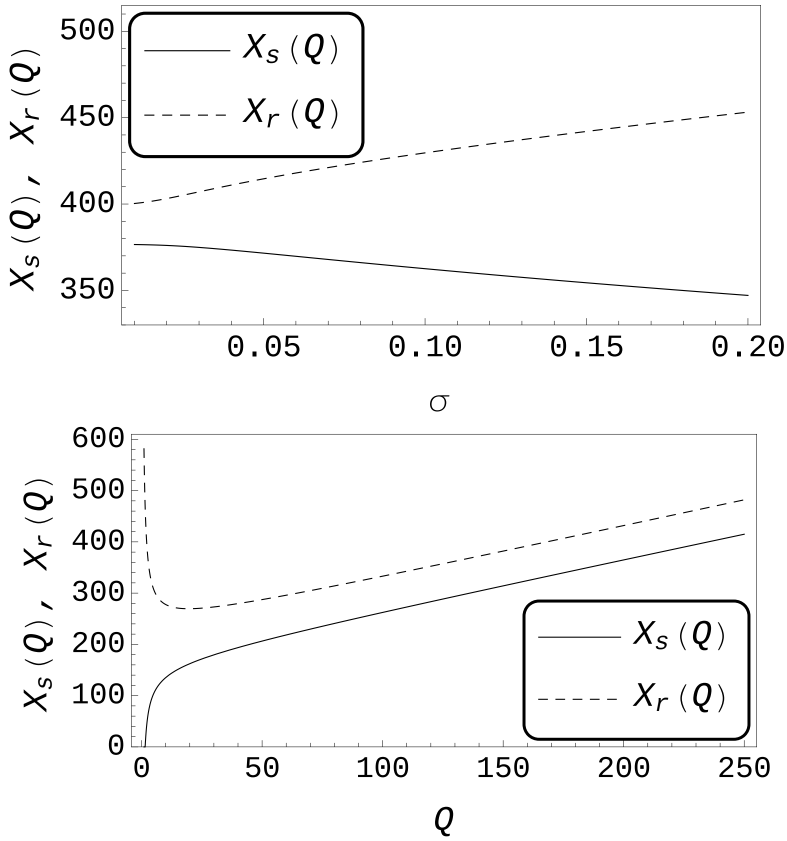

Figure 5.3: The effect of uncertainty σ (upper panel) and capacity choice Q (bottom panel) on the optimal operational triggers. Take parameter combination $Q = 197.81$ (for upper panel), $\sigma = 0.1$ (for bottom panel), $\rho = 0.1$, $\mu = 0.02$, $\eta = 1$, $S_d = S_s = 3000$, $\delta = 1000$, $\lambda = 0.7$, and $c = 200$.

Table 5.1 shows the optimal capacity and timing of investment for several levels of uncertainty. A model that is closely related to our work is the inflexible model of Hagspiel (2011), in which a firm has to decide when to undertake the capacity investment along with the capacity size[6]. In the inflexible model a suspension option prevents a firm to produce as soon as demand is such that price will fall below unit production costs. In order to compare our model with Hagspiel (2011), we have chosen the same parameter values. The optimal

[6]This model is inflexible compared to another model that Hagspiel (2011) defines, i.e. the flexible model, where a firm can also optimize production quantities.

investment strategy for our model, shown in Table 5.1, confirms the optimal investment strategy illustrated in Figure 2.4 in Hagspiel (2011).

σ	X_I^*	Q^*	$X_s(Q^*)$	$X_r(Q^*)$
0.1	464.74	197.81	362.57	429.62
0.15	678.30	383.19	547.45	616.33
0.20	1321.21	982.31	1141.31	1221.52

Table 5.1: Investment strategy and optimal operational triggers. Parameter values are ρ =0.1, μ =0.02, σ =0.1, $\eta = 1$, $c = 200$, $S_s = S_r$ =3000, δ =1000, and λ =0.7.

The last two columns of Table 5.1 show the effect of increasing uncertainty on the the switching triggers. In fact, these results can be explained by combining the two graphs in Figure 5.3 with the first two columns of Table 5.1. A higher level of uncertainty results in a larger capacity investment, which in turn increases both switching triggers. In effect, a firm speeds up the optimal moment to suspend production and delays the optimal moment to resume production (i.e. an indirect effect). Also, we have shown in Figure 5.3 that a higher level of uncertainty delays both the optimal moment to suspend as the optimal moment to resume operations (i.e. a direct effect). Table 5.1 shows that the indirect effect dominates the direct effect.

Our model departs from the inflexible model of Hagspiel (2011) by the introduction of positive switching costs. As an illustration, Figure 5.4 shows the scenario where $S_r = S_s = 0$, which corresponds to the inflexible model of Hagspiel (2011). Namely, a firm suspends and opens an operation when passing the level of uncertainty parameter $X_s(Q) = X_r(Q) = 397.8$. That is exactly the level of X_t for which the price is equal to the unit production costs. For positive switching costs, we find that $X_s(Q) \leq 397.8 \leq X_r(Q)$. Then, a firm shall not immediately suspend (open) operations when the profit turned negative (positive). It might rather have a small loss (lose some revenue from potential production) than pay the switching costs. Tsekrekos (2010) confirms this result. In Figure 5.4 we assume that the fixed level of capacity Q is optimal for $S_r = S_s$ =3000.

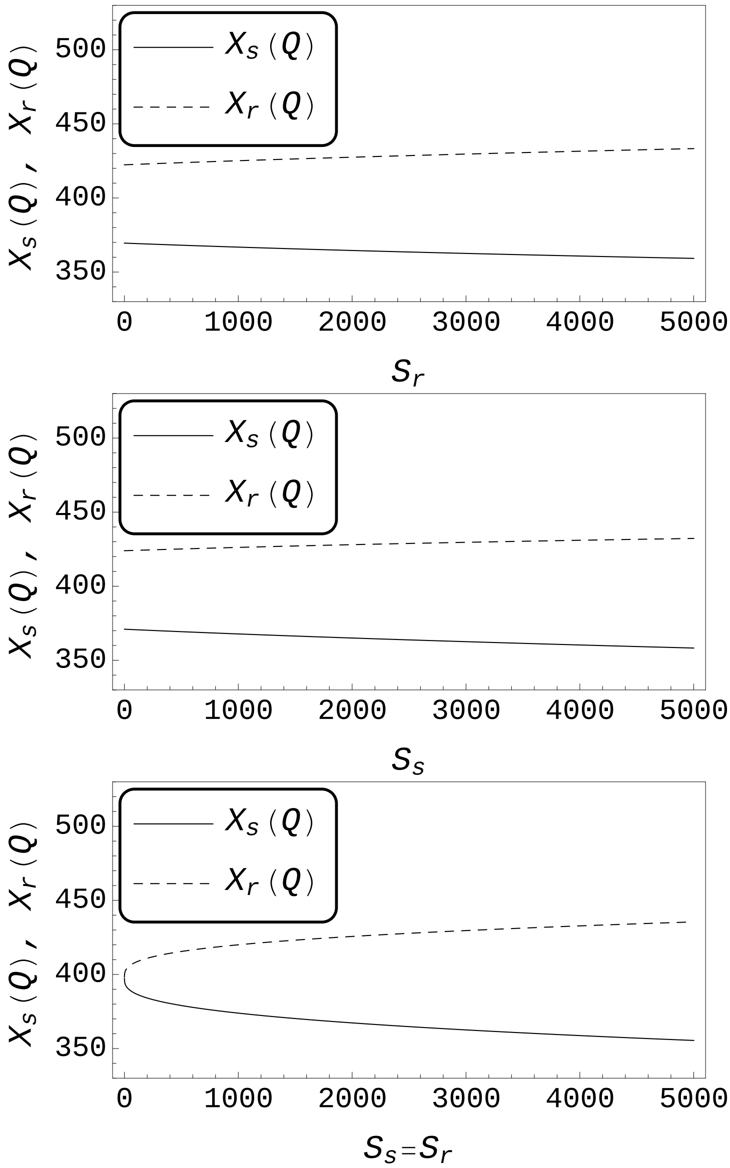

Figure 5.4: The effect of the switching cost parameters on the optimal operational triggers. Take parameter combination $Q = 197.81$, $\rho = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, $\eta = 1$, $\delta = 1000$, $\lambda = 0.7$, and $c = 200$. In the upper panel take S_s=3000, in the middle panel take S_r=3000.

5.4 Time-lag and Capacity Optimization in the Operational Flexible Model

In the previous section, a firm was given the opportunity to temporarily close down production for low prices. This section takes into account that the start-up of production is more involved. There is a time lag between the decision to resume and the start of the operations during which the firm is unable to obtain revenue from production. The suspension decision, on the other hand, can occur instantaneously. Contrary to Sødal (2006) and Bar-Ilan and Strange (1996) we do not consider extremely large time lags, motivated by the necessity to make an extensive investment at the moment the firm resumes the production. Instead, we assume that the old production location and equipment is still available after the restart.

In this section we see how the additional time lag changes the results from previous section.

5.4.1 Description of the Model

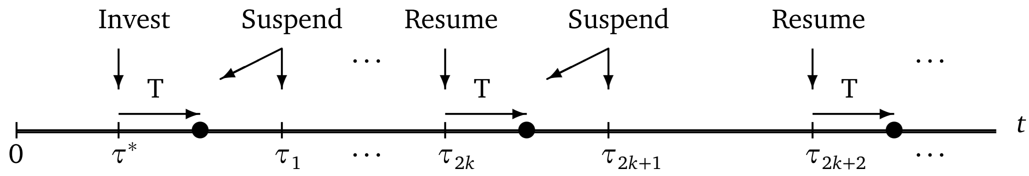

Figure 5.5: Decision-making time line for investing, suspending and resuming operations with time lags.

Figure 5.5 illustrates the decision-making time line with time lags. We start by looking at the scenario where a firm has just made the decision to resume operations at $t = \tau_{2k}$. There is a necessary preparation time before a firm can obtain the “old" profit flow. Denote the time that it takes to prepare for the restart of production, also known as the time lag, by T, i.e., the firm will be operating by time $t = \tau_{2k} + T$. If the last decision the firm made is the restarting decision, then the firm now owns the option to suspend operations at its optimal timing. It could be optimal to stop the production immediately after the time lag. In this case, the firm observes a price too low to continue production (i.e. $X_{\tau_{2k}+T} \leq X_{\tau_{2k+1}}$). Notice, this implies that the firm has not yet produced anything, it immediately goes back to the suspended state. Alternatively, the current price after the time lag is high enough to actually restart production (i.e. $X_{\tau_{2k}+T} > X_{\tau_{2k+1}}$), and a firm optimally supends production at time $t = \tau_{2k+1}$. After one such cycle has been completed, the firm will face similar decisions in the subsequent cycle. This is similar for all other previous and future operational decisions. Due to the stationarity in these decisions, we find that there is one mutual trigger for resuming operations and one mutual trigger for suspending operations. That is, $X_{\tau_{2k}} = X_r$ and $X_{\tau_{2k+1}} = X_s$, for all $k \in \mathbb{N}$.

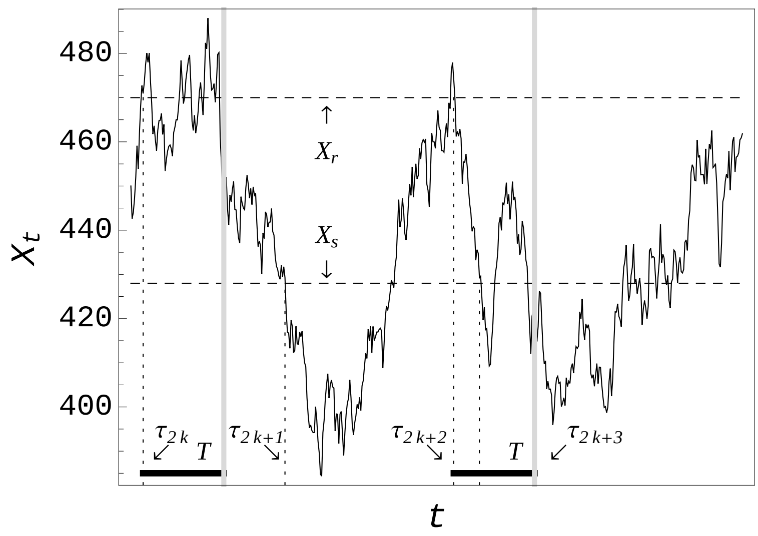

Figure 5.6: A simulated path of uncertainty parameter X_t. Assume resumption and suspension triggers $X_r = 470$ and $X_s = 428$.

Figure 5.6 illustrate the two possible scenarios for suspending operations. Assume that the firm has invested, and is initially inactive when starting to evaluate the price process. Almost immediately the firm makes the decision to resume the production. After the time lag, the firm produces for a while until it is optimal to suspend the production. Later, the firm resumes the operations again, but this time the price after the time lag is too low to actually produce, and production is suspended immediately.

Suspending Operations 5.4.2

In order to find the value functions and optimal decisions of the firm, we work backwards through the time line. Denote the expected net present value of a plant that has just resumed operations at time $t = \tau_{2k+2}$ by $F_r(X_r; X_s, X_r, Q)$ for some given $k \in \mathbb{N}$. By taking one step back in the decision-making time line, the next step is to look at the suspension decision of the firm. There are two opportunities for a firm to shut down the operation. A firm could immediately suspend after the time lag, i.e. at $t = \tau_{2k} + T$; in this case, it holds that $X_{\tau_{2k}+T} \leq X_s$. Alternatively, the firm optimally suspends operations at time $t = \tau_{2k+1}$. (To easy the notation, from now on we shall denote $X_{\tau_{2k}+T}$ by X_T.)

The net expected value of a plant that has just resumed production and now has the opportunity to suspend, is denoted by $F_s(X_T;X_s,X_r,Q)$ and is expressed as follows:

$$F_s(X_T;X_s,X_r,Q) = \begin{cases} \left(\frac{X_T}{X_r}\right)^{\beta_1} F_r(X_T;X_s,X_r,Q) - S_s - S_r & \text{if } X_T \leq X_s, \\ \frac{X_T Q}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho} - S_r + \left(\frac{X_T}{X_s}\right)^{\beta_2} \left(\frac{(\eta Q^2+cQ)}{\rho} - \frac{X_s Q}{\rho-\mu} \right. & \\ \qquad \left. - S_s + \left(\frac{X_s}{X_r}\right)^{\beta_1} F_r(X_r;X_s,X_r,Q)\right) & \text{if } X_T > X_s. \end{cases} \tag{5.4.1}$$

In case the suspension decision is immediately performed at time $t = \tau_r + T$ (upper line of expression (5.4.1)), the firm pays switching cost S_s for switching towards the suspended state and S_r for the previous decision to restart the operation. Namely, similar to Bar-Ilan and Strange (1996) and Sødal (2006), we assume that the entry costs are paid at the end of the time lag. In case the firm pursues production after the time lag (lower line of expression (5.4.1)), it obtains a revenue equal to $\frac{X_T Q}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho}$, before it suspends production at time $t = \tau_{2k+1}$. To ease the notation, we shall from now on denote $F_s(X_T;X_s,X_r,Q)$ by $F_s(X_T)$.

5.4.3 Resuming Operations

Since we are working backwards through the decision-making time line, next we look at the firm's decision to resume the operation at time $t = \tau_{2k}$, which is followed by a time lag T. As discussed earlier, time $t = \tau_{2k} + T$ is the first moment where the firm can decide to suspend the production.

The value of the option to resume the operation is in fact just the net discounted expected value of closing the operation at a later moment in time. This expectation is discounted back to the moment where the operating decision is made (at $t = \tau_{2k}$). $F_r(X_r;X_s,X_r,Q)$ denotes the value of the decision to resume the operation at time $t = \tau_{2k}$, and is given by:

$$F_r(X_r; X_s, X_r, Q) = e^{-\rho T}\mathbb{E}[F_s(X_T)|X_r]. \tag{5.4.2}$$

From now on, we shall denote $F_r(X_r; X_s, X_r, Q)$ by $F_r(X_r)$. The expected value of the option discounted to time $t = \tau_{2k}$ in expression (5.4.2) can be written as:

$$\begin{aligned}
\mathbb{E}[F_s(X_T)|X_r] = &\\
&\int_0^{X_s}\left(\left(\frac{X_T}{X_r}\right)^{\beta_1} F_r(X_r) - S_r - S_s\right) f(X_T|X_r)dX_T\\
+ &\int_{X_s}^{\infty}\left(\frac{X_T Q}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho} - S_r + \left(\frac{X_T}{X_s}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho}\right.\right.\\
&\left.\left.\quad - \frac{X_s Q}{\rho-\mu} - S_s + \left(\frac{X_s}{X_r}\right)^{\beta_1} F_r(X_r)\right)\right) f(X_T|X_r)dX_T,
\end{aligned} \tag{5.4.3}$$

where $f(X_T|X_r)$ denotes the conditional probability density function at time $t = \tau_{2k}$ of the price of electricity at time $t = \tau_{2k} + T$, when the production preparation is completed. Equation (5.4.3) can be rewritten as:

$$\begin{aligned}
\mathbb{E}[F_s(X_T)|X_r] = &\\
&\left(\frac{X_r}{X_r}\right)^{\beta_1} F_r(X_r)\Phi(\nu(X_s,X_r) - \beta_1\sigma\sqrt{T})e^{\rho T}\\
- &(S_s - S_r)\Phi(\nu(X_s,X_r)) + \frac{X_r Q}{\rho-\mu}\left(1-\Phi(\nu(X_s,X_r)-\sigma\sqrt{T})\right)e^{\mu T}\\
- &\left(\frac{(\eta Q^2+cQ)}{\rho} + S_r\right)(1-\Phi(\nu(X_s,X_r))) + \left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho}\right.\\
&\left. - \frac{X_s Q}{\rho-\mu} - S_s + \left(\frac{X_s}{X_r}\right)^{\beta_1} F_r(X_r)\right)\left(1-\Phi(\nu(X_s,X_r)-\beta_2\sigma\sqrt{T})\right)e^{\rho T},
\end{aligned} \tag{5.4.4}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $\nu(X_s,X_r)$ is defined by

$$\nu(X_s,X_r) = \frac{\log(X_s) - \log(X_r) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}. \tag{5.4.5}$$

Appendix 5.A.1 explains the derivation from the expected value in expression (5.4.3) towards the solution in expression (5.4.4). The following proposition explains that for T very close to zero, the model described in this section simplifies to the model of Section 5.3 (See Appendix 5.A.2 for the proof.).

Proposition 5.1

The value functions of the model that incorporate time lag T move towards the value functions without a time lag, if $T \to 0$.

5.4.4 Switching Triggers

Following the solution method of McDonald (2002), we substitute expression (5.4.4) into (5.4.2). Solving for $F_r(X_r)$ yields the expression $G_r(X_s;X_r,Q)$:

$$
\begin{aligned}
&G_r(X_s;X_r,Q) = \\
&\quad \frac{e^{-\rho T}}{1-\Phi\left(\nu(X_s,X_r)-\beta_1\sigma\sqrt{T}\right)-\left(1-\Phi(\nu(X_r)-\beta_2\sigma\sqrt{T})\right)\left(\frac{X_s}{X_r}\right)^{\beta_1}\left(\frac{X_r}{X_s}\right)^{\beta_2}} \\
&\quad \Bigg[-(S_r+S_s)\Phi(\nu(X_s,X_r)) + \tfrac{X_rQ}{\rho-\mu}\left(1-\Phi(\nu(X_s,X_r)-\sigma\sqrt{T})\right)e^{\mu T} \\
&\quad -\left(\tfrac{(\eta Q^2+cQ)}{\rho}+S_0\right)(1-\Phi(\nu(X_s,X_r))) + \left(\tfrac{X_r}{X_s}\right)^{\beta_2} \\
&\quad \left(\tfrac{(\eta Q^2+cQ)}{\rho} - \tfrac{X_sQ}{\rho-\mu} - S_s\right)\left(1-\Phi(\nu(X_s,X_r)-\beta_2\sigma\sqrt{T})\right)e^{T}\Bigg].
\end{aligned}
\tag{5.4.6}
$$

Once found a closed form expression for $G_r(X_s;X_r,Q)$, this is substituted into the value function of a closed operation, i.e.

$$
G_s(X_r;X_s,Q) = \left(\frac{X_s}{X_r}\right)^{\beta_1}(G_r(X_s;X_r,Q))). \tag{5.4.7}
$$

Notice that both value equations are a function of X_r and X_s. Next, we take the first-order condition of $G_r(X_s;X_r,Q)$, with respect to X_s, and the first order condition of $G_s(X_r;X_s,Q)$ with respect to X_r:

$$
\frac{\partial G_r(X_s;X_r,Q)}{\partial X_s} = 0,
$$

$$
\frac{\partial G_s(X_r;X_s,Q)}{\partial X_r} = 0.
$$

These equations are solved simultaneously for X_s and X_r for every $Q \geq 0$. Solutions are denoted by $X_s(Q)$ and $X_r(Q)$. We use the software program Mathematica to find a solution to this problem.

Investment Trigger and Capacity Choice 5.4.5

Suppose that the firm has not yet invested in the market. The option value of the firm that invests at X_I in capacity Q is given by

$$
\begin{aligned}
&F_I(X_t;X_I,Q) = \\
&e^{-\rho T}\left(\frac{X_t}{X_I}\right)^{\beta_1} \quad \Bigg(\left(\frac{X_T}{X_r(Q)}\right)^{\beta_1} F_r(X_r(Q))\Phi(\nu(X_s(Q),X_T)-\beta_1\sigma\sqrt{T})e^{\rho T} \\
&\qquad -(S_s-S_r)\Phi(\nu(X_s(Q),X_T)) \\
&\qquad +\frac{X_T Q}{\rho-\mu}\left(1-\Phi(\nu(X_s(Q),X_T)-\sigma\sqrt{T})\right)e^{\mu T} \\
&\qquad -\frac{(\eta Q^2+cQ)}{\rho}(1-\Phi(\nu(X_s(Q),X_T))) \\
&\qquad +\left(\frac{X_I}{X_s(Q)}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho}-\frac{X_s(Q)Q}{\rho-\mu}-S_s+\left(\frac{X_s(Q)}{X_r(Q)}\right)^{\beta_1}F_r(X_r(Q))\right) \\
&\qquad \left(1-\Phi(\nu(X_s(Q),X_T)-\beta_2\sigma\sqrt{T})\right)e^{\rho T}-\delta Q^{\lambda}\Bigg).
\end{aligned}
\tag{5.4.8}
$$

Also after the first decision to start operations, there exists a time lag to prepare the production process. Similar to what was explained in Section 5.3, the optimal capacity and investment trigger are found by taking the first order derivative of expression (5.4.8) with respect to X_I and Q. The optimal values are denoted by X_I^* and Q^*. Substitution of Q^* in $X_s(Q)$ and $X_r(Q)$ give $X_s(Q^*)$ and $X_r(Q^*)$.

Results 5.5

In order to compare the results from the model with a time lag (Section 5.4) to the model without a time lag (Section 5.3), we consider the set of parameters from Section 5.3. Another set of parameters that we could have considered is the same set parameters that Bar-Ilan and Strange (1996) and Sødal (2006) use. However, since they do not consider a demand function, the choice of η could determine the differences in results. Note that our model simplifies to the model of Bar-Ilan and Strange (1996) and Sødal (2006) by choosing η =0 and Q =1. Even though we take a slightly different version of the discount factor approach of Sødal (2006), we can replicate their results (Table 1, p. 1972) by making the same parameter assumptions, which can be seen as an extra correctness check for our model.

5.5.1 Value Functions

The value functions $V_s(X_t,X_s,X_r)$, $V_r(X_t,X_s,X_r)$ and $F_I(X_t;X_I,Q)$ are shown graphically in Figure 5.7, where

$$V_s(X_t;X_s,X_r) = \left(\frac{X_t}{X_r}\right)^{\beta_1} (F_r(X_r)) - \delta Q^{\lambda}, \tag{5.5.1}$$

$$\begin{aligned}
V_r(X_t;X_s,X_r) = & \\
& e^{-\rho T}\Bigg(\left(\tfrac{X_t}{X_r}\right)^{\beta_1} F_r(X_r)\Phi(\nu(X_s,X_t) - \beta_1\sigma\sqrt{T})e^{\rho T} \\
+ \quad & \tfrac{X_tQ}{\rho-\mu}\big(1-\Phi(\nu(X_s,X_t)-\sigma\sqrt{T})\big)e^{\mu T} - \Big(\tfrac{(\eta Q^2+cQ)}{\rho} + S_r\Big)(1-\Phi(\nu(X_s,X_t))) \\
+ \quad & -(S_s - S_r)\Phi(\nu(X_s,X_t)) + \left(\tfrac{X_t}{X_s}\right)^{\beta_2}\Big(\tfrac{(\eta Q^2+cQ)}{\rho} - \tfrac{X_sQ}{\rho-\mu} - S_s \\
& + \left(\tfrac{X_s}{X_r}\right)^{\beta_1} F_r(X_r)\Big) \\
& \quad \big(1-\Phi(\nu(X_s,X_t)-\beta_2\sigma\sqrt{T})\big)e^{\rho T}\Bigg) - \delta Q^{\lambda},
\end{aligned} \tag{5.5.2}$$

and $F_I(X_t;X_I,Q)$ as described in expression (5.4.8).

Notice that investment costs are subtracted from the operating and the closing value function, this in order to value match with the option value to invest $F_I(X_t;X_T,Q)$. Due to the assumption that the entry cost is paid after the time lag, there is a gap in the value of $S_r + S_s = 6000$ at $X_t = X_s$. However, when the uncertainty parameter X_t hits the trigger X_r, the operating and closing value function value match perfectly. The numerical example from Figure 5.7 assumes that there is no time lag.

It should be noted that under the assumption of a positive time lag, the gap in value at $X_t = X_s$ is smaller than $S_r + S_s$. Here, it becomes possible for a firm to suspend the operation right after the time lag. This is where total switching costs $S_r + S_s$ are immediately paid. Alternatively, it is optimal for the firm to pursue the production, and it just pays entry cost S_r immediately after the time lag. At a later moment, when it is optimal to suspend the operation, S_s is paid. The latter case describes the general case without a time lag. However, the expected discounted switching cost S_s is lower when paid right after the time lag, resulting in a slightly higher value of an active firm that has the option to suspend.

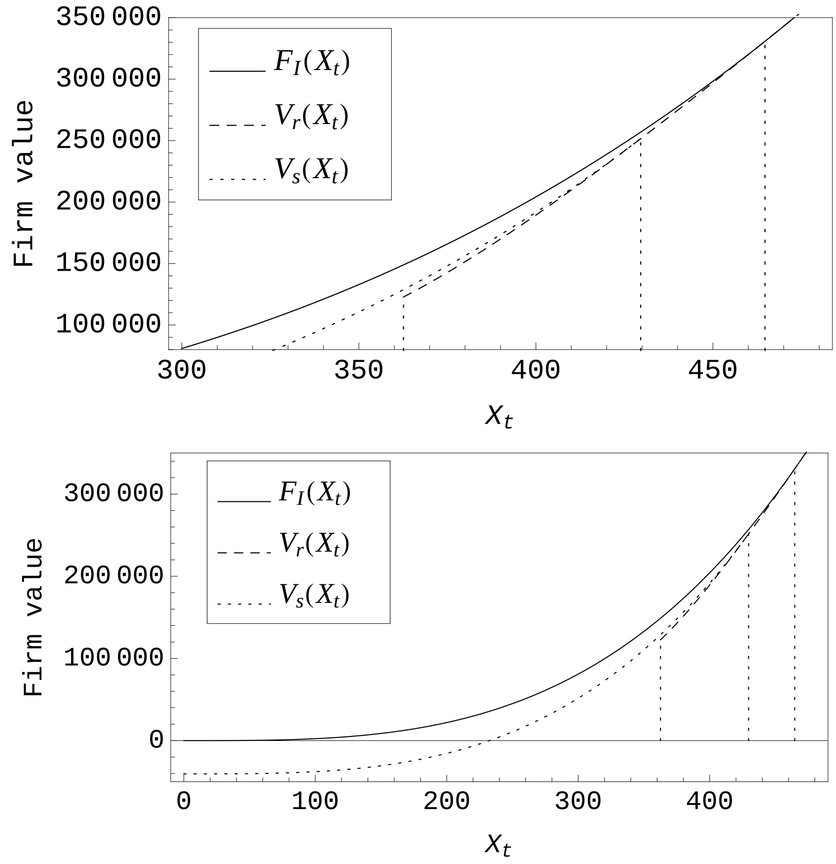

Figure 5.7: Value of the investment opportunity. Capacity $Q^* = 250.23$ and triggers $X_s = 362.6$, $X_r = 429.6$ and $X_I = 464.7$ are optimal under parameter combination $\rho = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, $\eta = 1$, $S_d = S_s = 3000$, $\delta = 1000$, $\lambda = 0.7$, $c = 200$, and $T = 0$.

Switching Triggers 5.5.2

Consider the situation of a firm that has invested and total capacity is known. Figure 5.8 illustrates the effect of uncertainty (bottom panel) and the length of the time lag (upper panel) on the optimal triggers to switch from closing to the operational state and vice versa. Superscript lag highlights the results that incorporate the time lag. The time lag speeds up the decision to resume the operation whenever a firm is in the suspended state but delays the optimal moment to suspend the operation.

First, consider the decision to resume the operation. The firm has the possibility to suspend the operation for low prices, however the upward potential

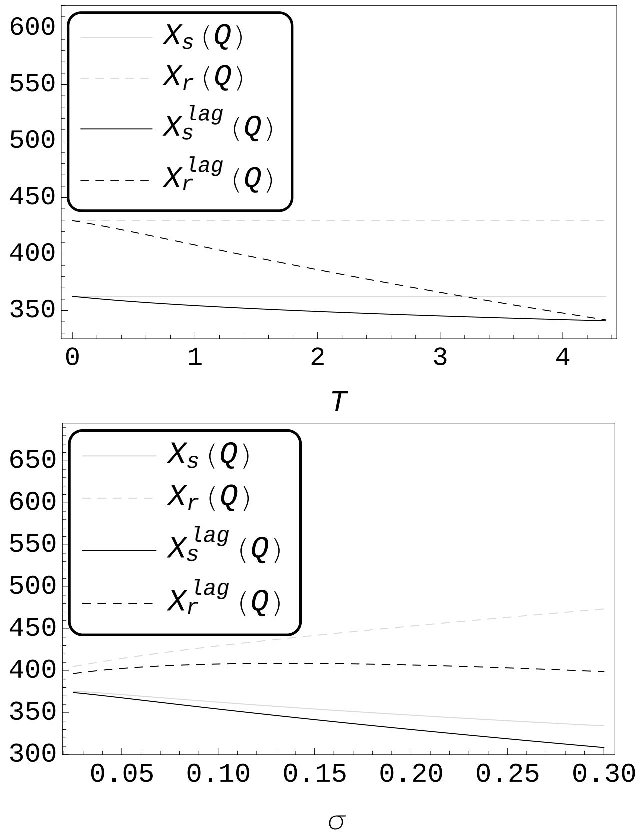

Figure 5.8: The effect of time lag T and uncertainty σ on the optimal triggers $X_r(Q)$, $X_s(Q)$, $X_r^{lag}(Q)$ and $X_s^{lag}(Q)$. Take parameter combination $Q = 197.81$, $\rho = 0.1$, $\mu = 0.02$, $\sigma = 0.1$, $\eta = 1$, $S_d = S_s = 3000$, $\delta = 1000$, $\lambda = 0.7$, $c = 200$, and $T = 1$. (Capacity choice Q is optimal for $T = 0$ and $\sigma = 0.1$.)

of the market is unlimited. Thus, the firm is expected to have a higher profit after the time lag. At the moment a firm decides to resume the operation, it has to wait for T time units before it actually gains revenue. As a result, it expects that after the time lag the price is already higher than would be optimal. Taking this into account, it is optimal for a firm to make the decision to resume production a little earlier. A positive market drift only strengthens this result[7]. Secondly, the time lag slightly delays the optimal moment to suspend the operation. Assume a firm that has to wait for a few months before it gains

[7] Also under the assumption of a negative market drift, the firm resumes the production at a lower trigger when it faces a larger time lag. However, this trigger does not diminish as much compared to the case of a positive market drift. Similarly, the trigger to suspend the production

revenue from the production facility. Once it is in production, it shall hesitate to suspend the production facility since it knows that this decision will be followed by another restart of the operation that includes an additional time lag.

We find that the time lag has a stronger influence on the moment to resume the operation than on the moment to suspend the operation. The operation is forced to be inactive for at least the duration of this time lag. Besides the switching costs that has to be paid from switching from one state to the other, the length of the time lag is another “burden" for the firm, because it cannot receive any (potential high) revenue in this period. Thus, for a very large time lag, a firm is not at all motivated to move towards the suspended state. For (unrealistic) large time lags, e.g. T=4.4 years, the switching triggers do not exist and the firm shall not use the opportunity to suspend the operation[8].

The bottom panel of Figure 5.8 confirms the result found in Bar-Ilan and Strange (1996) and Sødal (2006), i.e. a higher uncertainty might hasten a firms' entry decision. Bar-Ilan and Strange (1996) explain that there are two effects, where the level of uncertainty determines which one dominates. First, an increase in uncertainty raises the expected profit over the time lag period, which might result in an earlier entry. The expected profits can increase due to the abandonment option that truncates the downside of the market. Secondly, a higher level of uncertainty increases the likelihood of bad news, which delays the firm's decision to resume the production.

Investment Decision 5.5.3

We contribute to the literature by incorporating an initial investment decision where a firm has the additional option to optimize capacity. Table 5.2 shows for several levels of uncertainty the effect of the time lag on the optimal investment decisions and the switching triggers. The conventional result is that a larger capacity investment corresponds to a delay in the investment decision. This

does not increase as much for a larger time lag under a negative drift compared to the case with a positive drift.

[8]The switching levels X_s and X_r in Figure 5.8 are relatively large compared to Bar-Ilan and Strange (1996) and Sødal (2006) due to the higher assumed level of unit production choice. By definition of the geometric Brownian motion, we know that higher levels of X_t result in larger shocks. Therefore we find in our example, for a time lag larger than 4.4 years, already that a firm will not use the opportunity to enter and exit the market, where for the examples in the other two mentioned papers this result has not yet occurred after 8 years.

σ	T	X_I^{*lag}	Q^{*lag}	$X_s^{lag}(Q^{*lag})$	$X_s^{lag}(Q^{*lag})$
0.1	0	464.7	197.8	362.6	429.6
0.1	0.1	470.9	202.5	366.2	432.4
0.1	0.3	469.2	203.1	364.8	428.8
0.1	0.5	469.9	204.0	363.9	425.2
0.15	0	678.3	383.2	547.5	616.3
0.15	0.1	682.6	388.4	539.8	627.9
0.15	0.3	678.1	390.5	536.0	616.2
0.15	0.5	673.9	393.3	534.2	604.7
0.2	0	1321.2	982.3	1119.8	1247.1
0.2	0.1	1317.9	991.9	1113.2	1225.6
0.2	0.3	1295.9	1004.6	1106.9	1174.1
0.2	0.5	1274.7	1017.7	1106.5	1130.4

Table 5.2: Investment strategy and optimal operational triggers. Parameter values are ρ =0.1, μ =0.02, $\eta = 1$, $c = 200$, $S_s = S_r$ =3000, λ =0.7, and δ =1000.

result still holds when an increase in uncertainty is considered. However, where a larger time lag results in an increase in the size of investment, it does not necessarily lead to later entry. This result is further strengthened for a higher level of uncertainty. The larger capacity level is related to the expected profit of the firm over the time lag. Namely, the ability of a firm to suspend the production means that the downside of the investment is truncated. Therefore, the firm expects that the market to increase during the time lag, resulting in a larger expected profit once the firm can pursue the production after the delay. Thus, a larger time lag further strengthens this result, which justifies the larger capacity choice.

Result 5.1
Under capacity optimization, an increase in the length of the time lag results in a larger capacity size.

The effect of the time lag on the optimal timing of investment is ambiguous. Namely, it can be optimal to delay the investment moment, due the larger capacity choice. Alternatively, the firm hastens the investment decision, caused by the first time lag right after the investment decision. As explained for the upper panel of Figure 5.8, increasing the length of the time lag makes a firm hasten the entry decision, thus also the first entry decision.

Capacity optimization also affects the switching triggers after entry, as is shown in the last two columns of Table 5.2. From Figure 5.8 we know, when capacity optimization is not an issue, that a larger time lag reduces the level of both triggers, i.e. the direct effect of an increasing time lag. The switching triggers are also indirectly affected by the time lag, via the capacity level. We found that an increase in the length of the time lag results in a larger capacity level, which in turn increases the level of the switching triggers. Recall that a higher capacity level implies a higher level of X_t for which production becomes (un)profitable (see the bottom panel Figure 5.3). The indirect effect only dominates for a small initial level of the time lag and a relatively low uncertainty.

Result 5.2
Under capacity optimization, the firm chooses a higher investment trigger and higher exit and entry triggers, when uncertainty is relatively low and the initial time lag is small.

Counterfactual Effects 5.5.4

Another method to examine the effect of the time lag is to perform a counterfactual analysis. That is, assume that a firm neglects the time lag when it decides about the optimal triggers for closing and resuming the operation. It will use the triggers that do not incorporate the time lag. However, in reality there is a time lag, and therefore, its value is determined by the value functions that include this time lag. The counterfactual is the percentage loss in value when the firm does not suspend and resume the operation at the optimal triggers due to neglect of the time lag. The counterfactual value of resuming and suspending are given by

$$D_r(Q^*, Q^{*lag}) = \frac{V_r(X_s^{lag}(Q^{*lag}); X_s^{lag}(Q^{*lag}), X_r^{lag}(Q^{*lag})) - V_r(X_s^{lag}(Q^{*lag}); X_s(Q^*), X_r(Q^*))}{V_r(X_s^{lag}(Q^{*lag}); X_s(Q^*), X_r(Q^*))},$$

and

$$D_s(Q^*, Q^{*lag}) = \frac{V_s(X_r^{lag}(Q^{*lag}); X_s^{lag}(Q^{*lag}), X_r^{lag}(Q^{*lag})) - V_s(X_r^{lag}(Q^{*lag}); X_s(Q^*), X_r(Q^*))}{V_s(X_r^{lag}(Q^{*lag}); X_s(Q^*), X_r(Q^*))},$$

respectively. Here $V_s(X_t, X_s, X_r)$ and $V_r(X_t, X_s, X_r)$ are expressed by (5.5.1) and (5.5.2) respectively[9]. Table 5.3 includes the counterfactual values to the results in Table 5.2. Notice that we are only able to separately evaluate the two value functions of the firm.

σ	T	$D_s(Q^*, Q^{*lag})$	$D_r(Q^*, Q^{*lag})$
0.1	0	0%	0%
0.1	0.1	0.008%	0.002%
0.1	0.3	0.075%	0.023%
0.1	0.5	0.213%	0.068%
0.15	0	0%	0%
0.15	0.1	0.017%	0.009%
0.15	0.3	0.152%	0.088%
0.15	0.5	0.399%	0.240%
0.2	0	0%	0%
0.2	0.1	0.054%	0.042%
0.2	0.3	0.366%	0.298%
0.2	0.5	0.808%	0.673%

Table 5.3: Investment strategy, optimal operational triggers and counterfactual effects. Parameter values are ρ =0.1, μ =0.02, $\eta = 1$, $c = 200$, $S_s = S_r$ =3000, λ =0.7 and δ =1000.

As expected, the percentage loss in value due to the neglect of the time lag increases with the length of the time lag. When a firm resumes the production at the "wrong" (i.e. too high) trigger, it misses revenue due to the forced time lag. An increase in the market uncertainty also increases the counterfactual effects. This is indirectly caused by the increase in the optimal capacity choice. For a firm with a large production quantity, it is very costly to continue production too long against a negative price. Similarly, a firm misses out on a high profit, when the decision to continue the production is made later than optimal. Especially for a very uncertain market environment and a large time lag, it is important for a firm to incorporate the time lag in the model, in order to find the correct triggers. The counterfactual values may seem very low, however under a high

[9]Take $X_t = X_s^{lag}(Q^{*lag})$ for counterfactual value $D_r(Q^*, Q^{*lag})$, because the firm optimizes the option value to resume the operation at some moment in the future, under the assumption that it has just stopped producing. For similar reasons we take $X_t = X_r^{lag}(Q^{*lag})$ for counterfactual value $D_s(Q^*, Q^{*lag})$.

uncertainty the capacity level (and thus the production level) is such high that 0.8% of the total firm value is a big gain in revenue.

Since neither Bar-Ilan and Strange (1996) nor Sødal (2006) performed a counterfactual analysis for their parameter values, we perform this analysis in Table 5.4. The style of Table 5.4 is similar to the table that is presented in Bar-Ilan and Strange (1996) and Sødal (2006), therefore it considers σ^2 rather than σ. Our model can be simplified to their model by posing the additional assumptions $Q = 1$ and $\eta = 0$[10].

σ^2	$T = 0$		$T = 6$			
	X_s	X_r	X_s^{lag}	X_r^{lag}	$D_s(\cdot)$	$D_r(\cdot)$
0.00	1.000	1.025	1.000	1.025	0.000%	0.000%
0.01	0.834	1.243	0.793	1.146	1.109%	1.268%
0.02	0.795	1.312	0.736	1.151	1.373%	1.406%
0.03	0.770	1.362	0.697	1.149	1.526%	1.494%
0.04	0.751	1.405	0.666	1.145	1.632%	1.555%
0.05	0.735	1.442	0.640	1.140	1.710%	1.600%
0.10	0.682	1.586	0.551	1.112	1.921%	1.712%
0.20	0.623	1.791	0.450	1.072	2.043%	1.742%
0.30	0.587	1.953	0.388	1.048	2.045%	1.700%
0.40	0.560	2.094	0.342	1.036	2.003%	1.635%
0.50	0.539	2.221	0.308	1.031	1.941%	1.562%
0.60	0.522	2.338	0.280	1.033	1.868%	1.488%
0.80	0.495	2.554	0.237	1.049	1.712%	1.344%
1.00	0.474	2.753	0.206	1.078	1.556%	1.210%

Table 5.4: Investment strategy, optimal operational triggers and counterfactual effects. Parameter values are ρ =0.025, μ =0, $\eta = 0$, $c = 1$, $S_s = 0$, S_r =1, and $Q = 1$.

Contrary to the model where capacity is optimized, we find that there is a parabolic effect in the counterfactual values. For intermediate levels of market uncertainty, the firm has the most benefit from correctly choosing the triggers for which it resumes or suspends the operation. Namely, for relatively low levels of uncertainty, the firm is more confident that the price after the time lag is still high enough to pursue the production, and for very high levels of uncertainty,

[10]Bar-Ilan and Strange (1996) consider the same set of parameters, however Sødal (2006) detects a small technical error in this analysis of Bar-Ilan and Strange (1996) and rectifies the results. Similar to Sødal (2006), we shall give the counterfactual values for several values of the variance, rather than the standard deviation as assumed in earlier analysis

the price level is such unpredictable that the exact switching parameter matters less. The price is expected to reach a large bandwidth, within a short time frame anyway. However, we saw that when the firm is given the opportunity to optimize its capacity level, a higher level of uncertainty corresponds to a steep increase in the capacity level. When a large capacity level is involved, it contributes to the importance to choose the correct triggers, resulting in a larger counterfactual effect.

5.6 Conclusion

We extent the literature on entry and exit decisions by giving the firm the opportunity to optimize the size of its capacity at the moment that it invests in the market. Capacity is assumed to be lumpy. Therefore, after each entry decision, it uses the capacity that it initially invested in. We assume that a restart of an operation cannot occur instantaneously. Namely, it takes time to find new employees and skill them to the level of the old employees before suspending the firm. Besides the obvious example of a production facility, we can also think about a company that provides services, which needs to freeze or merge some divisions in economical hard times. Reopening a division requires employing some skilled employees which are able to learn the trick of the trade to the other newly hired employees. This indicates that it takes time before the (production)process is set in motion.

We find that the additional opportunity of a firm to optimize capacity changes the results to great extend. That is, a slightly higher level of uncertainty results in a delay in the optimal moment to resume the operation and hastens the supsending decision. The latter differs from the analysis without capacity optimization (see e.g. Bar-Ilan and Strange (1996) and Sødal (2006)). They find the conventional result that a firm delays the decision to suspend the operation under a higher level of uncertainty. Under capacity optimization, the firm increases the size of its capacity investment, and thereby also a larger level of the price intercept is required for a revenue flow equal to zero. Due to positive switching costs, the firm switches from the operational state towards the suspended state for a slightly negative revenue flow, which will occur later under a higher capacity level.

Furthermore, we find that the length of the time lag positively affect the size of capacity. Namely, the ability of a firm to suspend the production means that the downside of the investment is truncated. Therefore the presence of a time lag causes an increase in the expected profit over the time lag period, under the assumed uncertainty. Thus, a larger time lag also positively affects the expected profit, and therefore justifies the larger capacity choice.

Under the assumption of a fixed capacity, we confirm Bar-Ilan and Strange (1996)'s result that a larger time lag results in lower switching triggers. This results is not always true under capacity optimization. Namely, besides the mentioned direct effect, the triggers are also indirectly affected by an increase in the length of the time lags. A slightly larger time lag results in a larger capacity choice, resulting in higher switching triggers. This indirect effect dominates for an initial small time lag.

We assumed the additive demand structure, because this enables us to compare our results with Dangl (1999) and Hagspiel (2011). However Boonman and Hagspiel (2014) highlight the differences in results between additive and multiplicative demand structure. Where for the additive demand structure an increase in uncertainty results in a explosive increase in the capacity level, this increase is more moderate for the multiplicative demand structure. In this chapter we find that a higher level of uncertainty results in a higher trigger value to suspend the operation, which is indirectly caused by the steep increase in capacity. We leave it to further investigation to find out whether these results still holds under multiplicative demand. Additionally, it would be interesting to see how the results are affected under a competitive environment.

5.A Appendix

5.A.1 Expected Value of the Option to Discharge

When parameter X_r follows a GBM, the distribution of X_T given X_r is log normal, i.e.

$$\log(X_T)|_{X_r} \sim N(\log(X_r) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T).$$

Let us assume that $z = \log(X_T)$, $g = \log(X_r) + (\mu - \frac{1}{2}\sigma^2)T$ and $s^2 = \sigma^2 T$, then

$$z \sim N(g, s^2).$$

We are interested in $\mathbb{E}[X_T^\beta | X_r] = \mathbb{E}[e^{z\beta}|X_r]$. This is in fact equal to the calculation of the moment generating function. However, when we only need this expectation for $X_T \in (0, X_s)$, then the βth moment of X_T is calculated as follows:

$$\mathbb{E}[e^{z\beta}|z < \log(X_s)] = \int_{-\infty}^{\log(X_s)} e^{z\beta} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{1}{2}\frac{(z-g)^2}{s^2}} dz.$$

Define $y = \frac{z-g}{s}$, which implies that $z = ys + g$. The substitution rule gives

$$\mathbb{E}[e^{z\beta}|z < \log(X_s)] = \int_{-\infty}^{\frac{\log(X_s)-g}{s}} e^{(ys+g)\beta} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{1}{2}y^2} (\frac{dz}{dy}) dy.$$

Because we know that $\frac{dz}{dy} = s$, we find

$$\mathbb{E}[e^{z\beta}|z < \log(X_s)] = e^{\beta g} \int_{-\infty}^{\frac{\log(X_s)-g}{s}} e^{ys\beta} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy.$$

Replacement of some terms give

$$\mathbb{E}[e^{z\beta}|z < \log(X_s)] = e^{\beta g} \int_{-\infty}^{\frac{\log(X_s)-g}{s}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-\beta s)^2} e^{\frac{1}{2}s^2\beta^2} dy,$$

$$\mathbb{E}[e^{z\beta}|z < \log(X_s)] = e^{\beta g + \frac{1}{2}s^2\beta^2} \int_{-\infty}^{\frac{\log(X_s)-g}{s}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-\beta s)^2} dy.$$

Finally, the integral is defined as a CDF of variable y that is normally distributed with expectation βs and variance 1. When we also substitute $y = \frac{z-g}{s}$ back in the integral, we find

$$\begin{aligned} \mathbb{E}[e^{z\beta}|z < \log(X_s)] &= \mathbb{E}[X_T^{\beta}|X_T < X_s] \\ &= e^{\beta g + \frac{1}{2}s^2\beta^2} \Phi(\tfrac{\log(X_s)-g}{s} - \beta s), \end{aligned}$$

thus

$$\mathbb{E}[e^{z\beta}|z < \log(X_s)] = \\ X_r^{\beta} e^{\frac{1}{2}\sigma^2\beta^2 T + (\mu - \frac{1}{2}\sigma^2)\beta T} \Phi(\frac{\log(X_s) - \log(X_r) - (\mu - \frac{1}{2}\sigma^2 T)}{\sigma\sqrt{T}} - \beta\sigma\sqrt{T}).$$

Since β_1 (β_2) is the positive (negative) root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - \rho = 0,$$

we find that $e^{\frac{1}{2}\sigma^2\beta^2 T + (\mu - \frac{1}{2}\sigma^2)\beta T} = e^{\rho T}$. Applying this result to several components of the integral in expression (5.4.3), we find:

$$\int_{X_s}^{\infty} \tfrac{X_T Q}{\rho-\mu} f(X_T|X_r) dX_T = \tfrac{X_r Q}{\rho-\mu} \left(1 - \Phi(v(X_s, X_r) - \sigma\sqrt{T})\right) e^{\mu T}$$

$$-\int_{X_s}^{\infty} \left(\tfrac{(\eta Q^2 + cQ)}{\rho} + S_r\right) f(X_T|X_r) dX_T = -\left(\tfrac{(\eta Q^2 + cQ)}{\rho} + S_r\right) (1 - \Phi(v(X_s, X_r)))$$

$$\int_0^{X_s} \left(\tfrac{X_T}{X_r}\right)^{\beta_1} (F_r(X_r)) f(X_T|X_r) dX_T = \left(\tfrac{X_r}{X_r}\right)^{\beta_1} (F_r(X_r)) \Phi(v(X_s, X_r) - \beta_1\sigma\sqrt{T}) e^{\rho T}$$

The solutions for the remaining components in expression (5.4.3) are obtained similarly. □

Proof Proposition 5.1 5.A.2

We show that expression (5.4.2) moves towards expression (5.3.1) for $T \to 0$. For this, we introduce a new notation to distinguish the option values for a model with a time lag from the option values without a time lag. $F_r(X_s, T)$ denotes the option value of a firm that has just made the decision to resume production and now obtains the option to suspend at an optimal moment after the time lag. For the model without a time lag, we define this option by $F_r(X_s, 0)$. We will show that $\lim_{T\to 0} F_r(X_s, T) = F_r(X_s, 0)$. Substituting (5.4.4) into (5.4.2) gives:

$$
\begin{aligned}
&F_r(X_r, T) = \\
&e^{\rho T}\Bigg(\quad \left(\tfrac{X_r}{X_r}\right)^{\beta_1} F_r(X_r, T)\Phi(\nu(X_s,X_r) - \beta_1\sigma\sqrt{T})e^{\rho T} \\
&\quad + \quad \tfrac{X_rQ}{\rho-\mu}\left(1-\Phi(\nu(X_s,X_r)-\sigma\sqrt{T})\right)e^{\mu T} - \left(\tfrac{(\eta Q^2+cQ)}{\rho} + S_r\right)(1-\Phi(\nu(X_s,X_r))) \\
&\quad + \quad -(S_s - S_r)\Phi(\nu(X_s,X_r)) + \left(\tfrac{X_r}{X_s}\right)^{\beta_2}\Big(\tfrac{(\eta Q^2+cQ)}{\rho} - \tfrac{X_sQ}{\rho-\mu} - S_s \\
&\qquad + \left(\tfrac{X_s}{X_r}\right)^{\beta_1} F_r(X_r, T)\Big)\left(1-\Phi(\nu(X_s,X_r)-\beta_2\sigma\sqrt{T})\right)e^{\rho T}\Bigg).
\end{aligned}
\tag{5.A.1}
$$

Since $X_r \geq X_s$, we find that $\nu(X_s,X_r) < 0$, where $\nu(X_s,X_r)$ is defined by expression (5.4.5). For $T \to 0$, it holds that $\nu(X_s,X_r) \to -\infty$.
Hence, $\lim_{T\to 0}\Phi(\nu(X_s,X_r)) = 0$ and $\lim_{T\to 0}\Phi(\nu(X_s,X_r) - \beta_1\sigma\sqrt{T}) = 0$, which eliminates two terms in expression (5.A.1). For similar reasons we find that $\lim_{T\to 0}\left(1-\Phi(\nu(X_s,X_r)-\sigma\sqrt{T})\right) = 1$, $\lim_{T\to 0}(1-\Phi(\nu(X_s,X_r))) = 1$ and $\lim_{T\to 0}\left(1-\Phi(\nu(X_s,X_r)-\beta_2\sigma\sqrt{T})\right) = 1$. In order to finish the proof we additionally have $\lim_{T\to 0} e^{\rho T} = 1$ and $\lim_{T\to 0} e^{\mu T} = 1$.

Hence, we find that:

$$
\begin{aligned}
F_r(X_r, 0) = {} & \frac{X_rQ}{\rho-\mu} - \frac{(\eta Q^2+cQ)}{\rho} - S_r \\
& + \left(\frac{X_r}{X_s}\right)^{\beta_2}\left(\frac{(\eta Q^2+cQ)}{\rho} - \frac{X_sQ}{\rho-\mu} - S_s + \left(\frac{X_s}{X_r}\right)^{\beta_1} F_r(X_r, 0)\right),
\end{aligned}
$$

which is similar to expression (5.3.1) with $X = X_r$, after substitution of expression (5.3.2) in which $X_t = X_s$. Notice that, due to the assumption that switching cost S_r is paid after the time lag, S_r is placed in a different position in the above formula than in expression (5.3.1). However, since $T \to 0$, this does not affect the results. □

Bibliography

Aguerrevere, F. L. (2003): Equilibrium investment strategies and output price behaviour: a real-options approach. *The Review of Financial Studies*, 16(4):1239–1272. Cited on pages 56 and 61.

Aguerrevere, F. L. (2009): Real options, product market competition and asset returns. *Journal of Finance*, 64(2):957–983. Cited on page 56.

Anand, K. S. and K. Girotra (2007): The strategic perils of delayed differentiation. *Management Science*, 53(5):697–712. Cited on pages 14, 15, 18, 32, 42, 62, 110, and 136.

Anupindi, R. and L. Jiang (2008): Capacity investment under postponement strategies, market competition, and demand uncertainty. *Management Science*, 54(11):1876–1890. Cited on pages 32, 57, 107, and 108.

Argenziano, R. and P. Schmidt-Dengler (2014): Clustering in N-player preemption games. *Journal of the European Economic Association*, 12(2):368–396. Cited on page 108.

Aspray, W., G. Royer, and M. G. Ocepek (2013): *Food in the Internet Age*. Springer International Publishing. Cited on pages 4 and 5.

Bai, G., S. Hsu, and R. Krishnan (2008): Accounting performance, cost structure, and firms capacity investment decision. *Working paper, Harvard Business School/ Michigan State University*. Cited on page 74.

Bar-Ilan, A. and W. C. Strange (1996): Investment lags. *American Economic Review*, 86(3):610–622. Cited on pages 7, 9, 10, 133, 134, 140, 144, 146, 149, 153, 157, 158, and 159.

Bar-Ilan, A. and W. C. Strange (1999): The timing and intensity of investment. *Journal of Macroeconomics*, 21(1):57–77. Cited on pages 59 and 108.

Bish, E. K. and Q. Wang (2004): Optimal investment strategies for flexible resources, considering the pricing and correlated demands. *Operations Research*, 52(6):954–964. Cited on page 13.

Boonman, H. J. and V. Hagspiel (2014): Sensitivity of Demand Function Choice in a Strategic Real Options Context. *Working paper, Tilburg University*. Cited on pages 55, 107, 108, 117, 120, 125, and 159.

Boonman, H. J., V. Hagspiel, and P. M. Kort (2014a): Dedicated vs Product Flexible Production Technology: Strategic Capacity Investment Choice. *Working paper, Tilburg University*. Cited on page 11.

Boonman, H. J., V. Hagspiel, and P. M. Kort (2014b): Optimal Capacity Investment in Oligopoly: a Real Options Approach. *Working paper, Tilburg University*. Cited on page 105.

Boonman, H. J. and A. Siddiqui (2014): Capacity Optimization in an Operational Model with Time-Lags. *Working paper, Tilburg University*. Cited on page 131.

Bouis, R., K. J. M. Huisman, and P. M. Kort (2009): Investment in oligopoly under uncertainty: the accordion effect. *International Journal of Industrial Organization*, 27(2):320–331. Cited on pages 3, 9, 58, 81, 105, 106, 107, 108, 110, 111, 124, 125, and 132.

Brennan, M. J. and E. S. Schwartz (1985): Evaluating natural resource investments. *Journal of Business*, 58(2):135–157. Cited on page 132.

Caves, R. E. and M. E. Porter (1977): From entry barriers to mobility barriers. *The Quarterly Journal of Economics*, 91(2):241–261. Cited on page 15.

Ceryan, O., O. Sahin, and I. Duenyas (2012): Dynamic pricing of substitutable products in the presence of capacity flexibility. *Operations Research*, 55(3):532–548. Cited on page 13.

Chang, M. (1993): Flexible manufacturing, uncertain consumer tastes, and strategic entry deterrence. *The Journal of Industrial Economics*, 41(1):77–90. Cited on page 14.

Chappell, L. (2005): Harbour: flexibility means survival. *Automotive news*. Cited on page 6.

Chevalier-Roigant, B., C. M. Flath, A. Huchzermeier, and L. Trigeorgis (2011): Strategic investment under uncertainty: a synthesis. *European Journal of Operational Research*, 25:639–650. Cited on pages 58 and 106.

Chod, J. and N. Rudi (2005): Resource flexibility with responsive pricing. *Operations Research*, 53(3):532–548. Cited on pages 13, 17, 18, 32, 62, 110, and 136.

Dangl, T. (1999): Investment and capacity choice under uncertain demand. *European Journal of Operational Research*, 117:415–428. Cited on pages 4, 57, 58, 59, 108, 133, 136, and 159.

Deneckere, R., H. P. Marvel, and J. Peck (1997): Demand uncertainty and price maintenance: markdowns as destructive competition. *American Economic Review*, 87(4):619–641. Cited on pages 18, 62, 110, and 136.

Dewit, G. and D. Leahy (2003): Rivalry in uncertain export markets: commitment versus flexibility. *Journal of International Economics*, 64(1):195–209. Cited on page 14.

Dixit, A. (1989): Entry and exit decision under uncertainty. *Journal of Political Economy*, 97(3):620–638. Cited on pages 133 and 134.

Dixit, A. (1991): A simplified treatment of the theory of optimal regulation of brownian motion. *Journal of Economic Dynamics and Control*, 15(4):657–673. Cited on page 132.

Dixit, A. K. and R. S. Pindyck (1994): *Investment under Uncertainty*. Princeton University Press. Cited on pages 106, 111, and 138.

Fine, C. H. and R. M. Freud (1990): Optimal investment in product flexible manufacturing capacity. *Management Science*, 36(4):449–466. Cited on page 13.

Fontes, B. M. M. (2008): Fixed versus flexible production systems: a real options analysis. *European Journal of Operational Research*, 188(1):169–184. Cited on page 57.

Forester, T. (1989): *High-Tech Society: The Story of the Information Technology Revolution*. The MIT Press. Cited on page 6.

Fudenberg, D. and J. Tirole (1985): Preemption and rent equilization in the adoption of new technology. *The Review of Economic Studies*, 52(3):383–401. Cited on pages 2, 106, and 113.

Goto, M., R. Takashima, M. Tsujimura, and T. Ohno (2008): Entry and exit decisions under demand uncertainty in a symmetric duopoly. *Working paper, Graduate School of Finance, Waseda University*. Cited on page 56.

Goyal, M. and S. Netessine (2007): Strategic technology choice and capacity investment under demand uncertainty. *Management Science*, 53(2):192–207. Cited on pages 8, 14, 15, 18, 35, 43, 62, and 110.

Goyal, M., S. Netessine, and T. Randall (2006): Deployment of manufacturing flexibility: an empirical analysis of the North American automotive industry. *working paper*. Cited on pages 6, 12, and 14.

Grenadier, S. R. (2000): Option exercise games: the intersection of real options and game theory. *Journal of Applied Corporate Finance*, 13(2):99–107. Cited on pages 56 and 58.

Hagspiel, V. (2011): *Optimal investment stragegies for product flexible and dedicated production systems under demand uncertainty*. Tilburg University Press. Cited on pages 4, 6, 17, 21, 32, 33, 57, 133, 136, 141, 142, and 159.

He, P., H. Ding, and Z. Hua (2011): Strategic choice of flexible production technology using game theory approach. *Robotics and Computer-Integrated Manufacturing*, 28(3):416–424. Cited on page 14.

Hoppe, H. C. (2000): Second-mover advantages in the strategic adoption of new technology under uncertainty. *International Journal of Industrial Organization*, 18(2):315–338. Cited on page 106.

Huisman, K. J. M. (2000): *Technology investment: a game theoretic real options approach*. Tilburg University Press. Cited on pages 69 and 70.

Huisman, K. J. M. and P. M. Kort (1999): Effects of strategic interactions on the option value of waiting. *Center Discussion Paper, Tilburg University*. Cited on page 3.

Huisman, K. J. M. and P. M. Kort (2004): Strategic technology adoption taking into account future technological improvements: a real options approach. *European Journal of Operational Research*, 159(3):705–728. Cited on page 106.

Huisman, K. J. M. and P. M. Kort (2014): Strategic capacity investment under uncertainty. *Center Discussion Paper, Tilburg University*. Cited on pages 56, 58, 62, 71, 72, 107, 108, and 117.

Kalyanaram, G., W. T. Robbinson, and G. L. Urban (1995): Order of market entry, established emperical generalizations, emerging emperical generalizations, and future research. *Marketing Science*, 14(3):212–221. Cited on pages 59, 70, and 76.

Kalyanaram, G. and D. R. Wittink (1994): Hetrogeneity in entry effects between nondurable consumer product categories. *International Journal of Research in Marketing*, 11(3):219–231. Cited on pages 59 and 77.

Karwowski, W. and M. Rahimi (1990): *Ergonomics of Hybrid Automated Systems II*. Elsevier, Amsterdam. Cited on page 13.

Kroll, D. E. and R. Wasden (1990): Personal versus Corporate Flexibility: A Paradigm. *working paper*. Cited on page 13.

Kulatilaka, N. and E. C. Perotti (1998): Strategic growth options. *Management Science*, 44(8):1021–1031. Cited on pages 56 and 58.

Lieberman, M. B. (1987): Market growth, economies of scale, and plant size in the chemical processing industries. *Journal of Industrial Economics*, 36(2):175–191. Cited on page 75.

Lieberman, M. B. and D. B. Montgomery (1988): First-mover advantages. *Strategic Management Journal*, 9:41–58. Cited on pages 60 and 70.

Mackintosh, J. (2003): Ford learns to bend with the wind. *Financial times*, February(14). Cited on pages 18 and 62.

Manne, A. S. (1961): Capacity expansion and probabilistic growth. *Econometrica*, 29:632–649. Cited on pages 57, 108, and 124.

Marx, R. N. (2007): An equilibrium model of investment under uncertainty. *The Review of Financial Studies*, 20(5):1461–1502. Cited on page 56.

Mason, R. and H. Weeds (2010): Investment, uncertainty, and pre-emption. *International Journal of Industrial Organization*, 28(3):278–287. Cited on page 106.

McDonald, R. and D. Siegel (1986a): The value of waiting to invest. *The Quartertly Journal of Economics*, 101(4):707–728. Cited on page 106.

McDonald, R. and D. R. Siegel (1986b): The value of waiting to invest. *The Quarterly Journal of Economics*, 101:707–727. Cited on page 132.

McDonald, R. L. (2002): *Derivatives Markets*. Addison Wesley. Cited on pages 133, 134, and 148.

Ming-Gao, X., L. Lin-lin, and W. Xiang-yang (2011): Capacity flexibility premium under different demand uncertainties. *International Conference on Management Science and Engeneering, Italy*. Cited on page 57.

Mossin, J. (1968): An optimal policy for lay-up decisions. *Swedish Journal of Economics*, 70:170–177. Cited on page 132.

Nishihara, M. and M. Fukushima (2008): Evaluation of firm's loss due to incomplete information in real investment decision. *European Journal of Operational Research*, 188(2):569–585. Cited on page 106.

Patel, P. C., S. Terjesen, and D. Li (2012): Enhancing effects of manufacturing flexibility through operational absorptive capacity and operational ambidexterity. *Journal of Operational Management*, 30(3):201–220. Cited on page 13.

Pawlina, G. and P. M. Kort (2006): Real Options in an assymmetric duopoly: who benefits from your competitive advantage. *Journal of Econometrics and Management Strategy*, 15(1):1–35. Cited on pages 106 and 108.

Rey, P. and B. Salanie (1990): Long-term, Short-term and Renegotiation: on the Value of Commitment in Contracting. *Econometrica*, 58(3):597–619. Cited on page 15.

Roller, L. H. and M. Tombak (1990): Strategic choice of flexible production technologies and welfare implications. *The Journal of Industrial Economics*, 38(4):417–431. Cited on page 14.

Ross, S. A., W. Westerfield, Randolph, and B. D. Jordan (2006): *Fundamentals of corporate finance, 7th ed.* The Mcgraw-Hill/Irwin Series in Finance, Insurance, and Real Estate. Cited on page 56.

Smets, F. (1991): Exporting versus FDI: The effect of uncertainty, irreversibilities and strategic interactions. *Working paper, Yale University, New Haven, Conneticut, United States of America*. Cited on pages 2 and 3.

Smit, J. T. J. and L. A. Ankum (1993): A real options and game-theoretic approach to corporate invetment strategy under competition. *Financial Management*, 22:241–250. Cited on pages 2 and 3.

Sødal, S. (2006): Entry and exit decisions based on a discount factor approach. *Journal of Economic Dynamics and Control*, 30(11):1963–1986. Cited on pages 7, 9, 134, 144, 146, 149, 153, 157, and 158.

Thijssen, J. J. J., K. J. M. Huisman, and P. M. Kort (2012): Symmetric equilibrium strategies in game theoretic real option models. *Journal of Mathematical Economics*, 48(4):219–225. Cited on page 106.

Tirole, J. (1988): *The theory of industrial organisation*. MIT Press, Cambridge, MA. Cited on pages 17, 36, and 58.

Tomlin, B. and Y. Wang (2005): On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing service Operations Management*, 7(3):37–57. Cited on page 13.

Trigeorgis, L. (1996): *Real Options: managerial flexibility and strategy in resource allocation*. The MIT Press, Cambridge, Massachusetts, USA. Cited on page 106.

Tsekrekos, A. E. (2010): The effect of mean reversion on entry and exit decisions under uncertainty. *Journal of Economic Dynamics and Control*, 34(4):725–742. Cited on page 142.

Tseng, M. C. (2004): Strategic choice of flexible manufacturing technologies. *International Journal of Production Economics*, 91(3):223–227. Cited on page 14.

Uran, G. L., T. Carter, S. Gaskin, and Z. Mucha (1986): Market share rewards to pioneering brands: an emperical analysis and strategic implications. *Management Science*, 32(6):645–659. Cited on pages 59 and 77.

Van Mieghem, J. A. (1998): Investment strategies for flexible resources. *Management Science*, 44(8):1071–1078. Cited on page 13.

Van Mieghem, J. A. (2003): Capacity management, investment, and hedging: review and recent developements. *Manufacturing and Service Operations Management*, 5(4):269–302. Cited on page 107.

Van Mieghem, J. A. and M. Dada (1999): Price versus production postponement: capacity and competition. *Management Science*, 45(12):1631–1649. Cited on pages 13 and 133.

Weeds, H. (2002): Strategic Delay in a Real Options Model of R&D Competition. *Review of Economic Studies*, 69:729–747. Cited on page 106.

Wu, J. (2007): Capacity preemption and leadership contest in a market with uncertainty. *Working paper, Mimeo, University of Arizona*. Cited on pages 107 and 108.

Yang, L., C. T. Ng, and T. C. E. Cheng (2011): Optimal production strategy under demand fluctuations: technology versus capacity. *European Journal of Operational Research*, 5(4):269–302. Cited on page 32.

Yang, M. and Q. Zhou (2007): Real options analysis for efficiency of entry deterrence with excess capacity. *Systems Engeneering - Theory and Practice*, 27:63–70. Cited on page 58.

Zhang, X., H. Huang, and X. Xia (2013): Oligopoly power producer's capacity investment model with contracts for differences. *Economic Theory*, 2013:1–13. Cited on page 108.